

## The Semantics of Determiners\*

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The study of generalized quantifiers over the past 15 years has enriched enormously our understanding of natural language determiners (Dets). It has yielded answers to questions raised independently within generative grammar and it has provided us with new semantic generalizations, ones that were basically unformulable without the conceptual and technical apparatus of generalized quantifier theory. Here we overview results of both these types.

**historical note** It was Montague (1969) who first interpreted natural language NPs as generalized quantifiers (though this term was not used by him). But it was only in the early 1980's with the publication of B&C (Barwise and Cooper, 1981) that the study of natural language Dets took on a life of its own. Also from this period are early versions of K&S (Keenan and Stavi, 1986) and Higginbotham and May (1981). The former fed into subsequent formal studies such as van Benthem (1984, 1986) and Westerståhl (1985). The latter focussed on specific linguistic applications of binary quantifiers, a topic initiated in Altham and Tennant (1974), drawing on the mathematical work of Mostowski (1957), and pursued later in a more general linguistic setting in van Benthem (1989) and Keenan (1987b, 1992). Another precursor to the mathematical study of generalized quantifiers is Lindström (1969) who provides the type notation used to classify quantifiers in many later studies.

Since these beginnings work on the semantics of Dets has proliferated, both empirically and mathematically. Westerståhl (1989) provides an historical overview up to 1987. Some important collections of articles are: van Benthem and ter Meulen (1985), Gärdenfors (1987), and van der Does and van Eijck (to appear). From a more linguistic perspective we note Lappin (ed., 1988a) and ter Meulen and Reuland (1987). K&W (Keenan and Westerståhl, to appear) is a recent overview relating the natural language studies and concurrent work in mathematical logic.

### 1. Background notions and terminology

Terminology first: In the S in (1), we call *work hard* a (*tensed*) *one place predicate* or  $P_1$ ,

(1) Most students work hard

*most students* is a *noun phrase* or NP, *student(s)* is a (*common*) *noun* or N, and *most* is a (*one place*) *Determiner* or  $Det_{(1)}$ . So we think of a  $De_1$  as combining with an N to make an NP, the latter combining with  $P_1$ s to make Ss.

Semantically we interpret Ss like (1) as true (**T**) or false (**F**) in a given situation (state of affairs). A situation  $s$  consists, in part, of *universe*  $E_s$ , the set of (possibly abstract) objects under discussion in  $s$ ; we think of tense marking on  $P_1$ s as giving us some information about the situation we are to interpret the sentence in. Given a situation  $s$ , we interpret  $P_1$ s as subsets (called *properties*) of  $E_s$  and we interpret NPs as *generalized quantifiers* (GQs), that is as functions from properties to truth values (possible sentence interpretations). Using upper case

bold for interpretations (in a situation  $s$ ), the truth value of (1) is given by

(2) **(MOST STUDENTS)(WORK HARD)**

That is, the truth value (**T** or **F**) which (1) is interpreted as in  $s$  is the one the function **MOST STUDENTS** maps the set **WORK HARD** to. (An equivalent formulation common in the literature: interpret *most students* as a set of properties and interpret (1) as **T** if the set **WORK HARD** is an element of that set).

Now the denotation of *most students* is built from those of *most* and *student*. And given a universe  $E$ , Ns like *student* (as well as *tall student*, *student who Mary likes*, etc.) are, like (tenseless)  $P_{1s}$ , interpreted as properties over  $E$  (= subsets of  $E$ ). So Dets like *most* can be represented as functions from  $P_E$ , the set of properties over  $E$ , into  $GQ_E$ , the set of generalized quantifiers over  $E$  (We usually suppress the subscript  $E$  when no confusion results).

We illustrate the interpretation of some Dets. Let  $E$  be given and held constant throughout the discussion. Consider **EVERY**, the denotation of *every*. We want to say that *Every student is a vegetarian* is (interpreted as) true, **T**, iff each object in the set **STUDENT** is also in the set **VEGETARIAN**. Generalizing,

(3) For all properties  $A, B$  **EVERY(A)(B) = T** iff  $A \subseteq B$

What (3) does is define the function **EVERY**. Its domain is the collection  $P_E$  of subsets of  $E$  and its value at any  $A$  in  $P_E$  is the GQ **EVERY(A)** - namely, that function from properties to truth values which maps an arbitrary property  $B$  to **T** if and only if  $A$  is a subset of  $B$ . Here are some other simple cases which employ some widely used notation:

(4) a. **NO(A)(B) = T** iff  $A \cap B = \emptyset$

Here  $\emptyset$  is the empty set and (4a) says that *No A's are B's* is true iff the set of things which are members of both  $A$  and  $B$  is empty.

b. **(FEWER THAN FIVE)(A)(B) = T** iff  $|A \cap B| < 5$

c. **(ALL BUT TWO)(A)(B) = T** iff  $|A - B| = 2$

Here  $A - B$  is the set of things in  $A$  which are not in  $B$ , and in general for  $C$  a set,  $|C|$  is the cardinality of  $C$ , that is, the number of elements of  $C$ . So (4b) says that *All but two A's are B's* is true iff the number of things in  $A$  which are not in  $B$  is exactly 2.

d. **(THE TEN)(A)(B) = T** iff  $|A| = 10$  and  $A \subseteq B$

This says e.g. that *The ten children are asleep* is true iff the number of children in question is 10 and each one is asleep.

- e. **NEITHER**(A)(B) = **T** iff  $|A| = 2 \ \& \ A \cap B = \emptyset$   
 f. **MOST**(A)(B) = **T** iff  $|A \cap B| > |A - B|$

Here we have taken *most* in the sense of *more than half* .  $\square$

To test that the definitions above have been properly understood the reader should try to fill in appropriately the blanks in (5).

(5) **(MORE THAN FOUR)**(A)(B) = **T** iff \_\_\_\_\_

**BOTH**(A)(B) = **T** iff \_\_\_\_\_

**(EXACTLY TWO)**(A)(B) = **T** iff \_\_\_\_\_

**(JUST TWO OF THE TEN)**(A)(B) = **T** iff \_\_\_\_\_

**(LESS THAN HALF THE)**(A)(B) = **T** iff \_\_\_\_\_

**(BETWEEN FIVE AND TEN)**(A)(B) = **T** iff \_\_\_\_\_

Finally, Keenan and Moss (1985) extend the class of Dets to include two place ones such as *more...than...* which they treat as combining with two Ns to form NPs like *more students than teachers*. Such expressions have the basic distribution of NPs: they occur as subjects (6a), objects (6b), objects of prepositions (6c); they occur in ECM = "raising to object" constructions (6d), and they move under passive (6d).

- (6) a. More students than teachers came to the party  
 b. John knows exactly as many students as teachers  
 c. Mary has talked with fewer students than teachers  
 d. We believe more students than teachers to have signed the petition  
 e. More students than teachers are believed to have signed the petition

We may correctly interpret NPs like *more students than teachers* as the value of the Det<sub>2</sub> function **(MORE...THAN...)** at the pair <**STUDENT,TEACHER**> of properties given as follows (writing **MORE A THAN B** instead of **(MORE...THAN...)(A)(B)**):

(7) For all properties A,B,C **(MORE A THAN B)**(C) = **T** iff  $|A \cap C| > |B \cap C|$

Again on this pattern the reader should be able to define the functions in (7). See Beghelli (1992, 1993) for much more extensive discussion of cardinal comparatives.

(8) **FEWER...THAN..., FIVE MORE...THAN..., EXACTLY AS MANY...AS..., MORE THAN TWICE AS MANY...AS..., THE SAME NUMBER OF...AS...**

## 2. Two Types of Generalizations

One type of generalization we will be concerned with involves characterizing linguistically significant classes of NPs in terms of the Dets used to build them. (So here semantic work converges with such syntactic work as Abney (1987), Stowell (1987, 1991) and Szabolcsi (1987), which takes Dets as the "heads" of expressions of the form [Det + N]). For example, which NPs occur naturally in the *pos of* position in partitives is significantly determined by the choice of Det:

- (9) a. Two of the/these/John's cats                      b. \*Two of no/most/few cats

Changing the N from *cats* to *students* or *pictures that John took* does not change grammaticality in (9a) or (9b), but changing the Dets may. Similarly which NPs occur naturally in Existential There contexts, (10a,b), and which license negative polarity items in the predicate, (11a,b), are significantly determined by the choice of Det.

- (10) a. There aren't more than ten boys in the roo  
b. \*There aren't most boys in the roo

- (11) a. Fewer than five students here have ever been to Pinsk  
b. \*Some students here have ever been to Pinsk

Queries like those in (12) determine a second type of Det based generalization:

- (12) a. Are there constraints on which functions from properties to generalized quantifiers can be denoted by natural language Dets?  
b. Do lexical (= syntactically simple) Dets satisfy stronger constraints on their possible denotations than syntactically complex ones?

Questions like these arise naturally within generalized quantifier theory, but they also have a natural interpretation in a more classical linguistic setting. An affirmative answer to (12a) limits the task faced by the language learner and thus helps account for how the semantic system is learned with limited exposure to imperfect data. An affirmative answer to (12b) is even more interesting from the learning theory perspective. Modulo idioms, syntactically complex expressions are interpreted as a function of their parts. Thus if we know how a complex expression is built and we know what its parts mean we can figure out what the entire expression means. So a significant part of the learning problem in semantics reduces to the learning of the meanings of lexical items (including grammatical morphology).

Both the types of questions we raise push us to consider as large a class of Dets as possible. The more Dets we consider the more NPs we classify and the more significant are claims concerning constraints on Det denotations.

Let us first then overview the classes of Dets we consider. The classes overlap and are only

informally given; later we provide several precisely defined subclasses of Dets. Our purpose is to make the reader aware of the diversity of NP and Det types we generalize over. Some linguists would prefer to analyze some of our complex Dets differently. But eliminating some of our examples preserves the generalizations we make, since they hold for the larger class. Delimiting the class too narrowly by contrast runs the risk that our generalizations will be vitiated when new examples are added. Our discussion of Dets draws extensively on K&S.

### 3. Some Types of Determiners in English

#### **Lexical Dets**

every, each, all, some, a, no, several, neither, most, the, both, this, my, these, John's, ten, a few, a dozen, many, few

#### **Cardinal Dets**

exactly/approximately/more than/fewer than/at most/only ten, infinitely many, two dozen, between five and ten, just finitely many, an even/odd number of, a large number of

#### **Approximative Dets**

approximately/about/nearly/around fifty, almost all/no, hardly any, practically no

#### **Definite Dets**

the, that, this, these, my, his, John's, the ten, these ten, John's ten

#### **Exception Dets**

all but ten, all but at most ten, every...but John, no...but Mary,

#### **Bounding Dets**

exactly ten, between five and ten, most but not all, exactly half the, (just) one...in ten, only SOME (= some but not all; upper case = contrastive stress), just the LIBERAL, only JOHN's

#### **Possessive Dets**

my, John's, no student's, either John's or Mary's, neither John's nor Mary's

#### **Value Judgment Dets**

too many, a few too many, (not) enough, surprisingly few, ?many, ?few

#### **Proportionality Dets**

exactly half the/John's, two out of three, (not) one...in ten, less than half the/John's, a third of the/John's, ten per cent of the/John's, not more than half the/John's

#### **Partitive Dets**

most/two/none/only some of the/John's, more of John's than of Mary's, not more than two

of the ten

#### **Negated Dets**

not every, not all, not a (single), not more than ten, not more than half, not very many not quite enough, not over a hundred, not one of John's

#### **Conjoined Dets**

at least two but not more than ten, most but not all, either fewer than ten or else more than a hundred, both John's and Mary's, at least a third and at most two thirds of the, neither fewer than ten nor more than a hundred

### Adjectively Restricted Dets

John's biggest, more male than female, most male and all female, the last...John visited, the first ...to set foot on the Moon, the easiest...to clean, whatever...are in the cupboard

The three dots in the expressions above indicate the locus of the N argument. E.g. in *not one student in ten* we treat *not one...in ten* as a discontinuous Det. In general we have two prima facie reasons for positing discontinuous analyses: One, often the N + postnominal material has by itself no reasonable interpretation and so is not naturally treated as a constituent. Thus if we analyzed *not one student in ten* as [*not one*[*student in ten*]] we should have to assign a meaning to *student in ten*, but this string seems meaningless. So does *student but John* in *no student but John* and *man to set foot on the Moon* in *the first man to set foot on the Moon*.

And two, often the presence of the postnominal material and the prenominal material are not independent. If the N + postnominal material, such as *student in ten* above, were a constituent it would have to be able to combine with Dets (e.g. *not one*) prenominally. But in fact the choice of Det is highly constrained. How would we block *\*the/this/John's student in ten* and *\*the/this/Mary's/one student but John*?

Our point here is simply that there are some sensible reasons (K&S) for treating the complex expressions above as Dets. Our analysis is certainly not without problems of its own (Lappin 1988b and Rothstein 1988) and very possibly some of our cases will find a non-discontinuous analysis (see Moltmann, to appear and von Stechow, 1993 on exception Dets).

Finally we note some further candidates for two (and  $k > 2$ ) place Dets. The most natural cases are the cardinal comparatives like *more..than..* and *as many...as...* mentioned in (6) and (8). But Keenan & Moss (1985), the most extensive discussion of k-place Dets, suggest for example a 2 place analysis of *every...and...* in (13).

- (13) a. every man and woman jumped overboard  
b. **(EVERY...AND...)(MAN,WOMAN)(JUMPED OVERBOARD)**  
c. **(EVERY...AND...)(A,B)(C) = T** iff **EVERY(A)(C) = T** and **EVERY(B)(C) = T**

On this analysis (13a) would be true iff every man jumped overboard and every woman jumped overboard, in fact the natural interpretation of (13a). Possibly (13a) has another interpretation on which it means that everyone who was both a man and a woman jumped overboard. This is accommodated by treating *man and woman* as conjoined Ns, combined then with the Det<sub>1</sub> *every*. Such an analysis is less implausible in *every author and critic*. One advantage of the 2 place analysis of *every...and...* is that it generalizes naturally to  $k > 2$  place Dets. In *every man, woman and child* we allow that *every...and...* has combined directly with three Ns. So we might treat *every...and...* as a Det of variable arity, combining with  $k \geq 1$  Ns to form an NP and taking the form *every* in the case  $k = 1$ .

Before turning to our promised generalizations we note first one restriction on the Dets we generalize about. Namely, we limit ourselves to Dets that are *extensional* defined by:

- (14) A Det<sub>1</sub> D is *extensional* iff for all common noun phrases N,N' if N and N' are interpreted as the same set in a situation s then [D + N] and [D + N'] are interpreted as the same GQ in s.

One sees that *not enough* and *too many* are not extensional: We can imagine a situation in which the doctors and the lawyers are the same individuals but *Not enough doctors attended the meeting* is true and *Not enough lawyers attended the meeting* is false (say we need a hundred doctors for a quorum and just one lawyer for legal purposes, and just 95 doctor-lawyers show up). Judgments concerning the interpretation and extensionality of *many* and *few* are problematic in the literature. K&S argue that they are not extensional but B&C, Westerståhl (1985) and Lappin (1988b) attempt more extensional treatments, though their proposals are not identical. In this paper we shall largely exclude *many* and *few* from the generalizations we propose since our judgments regarding their interpretations are variable and often unclear. Comparable problems obtain for Det<sub>2</sub>s when *many* modifies them, as in *many more students than teachers [came to the party]*.

#### 4. Generalizations about English Determiner

Let us write DNP (*Determined NPs*) for NPs built from a Det and an appropriate number of Ns. Some semantic properties of DNPs are determined by the semantic nature of their Ns and other of their properties are determined by the Det.

For example, whether a DNP is animate, human or female is determined by the nature of its Ns. More generally, whether a DNP satisfies the selectional restrictions of a predicate is determined by its Ns. Thus *#Every ceiling laughed* is bizarre (here noted #) since ceilings are not the kinds of things that can laugh. And the judgment doesn't change if *every* is replaced by *most of John's* or *at least two but not more than ten*.

For DNPs built from Det<sub>2</sub>s both Ns are relevant (Keenan, 1987c). Thus *Fewer girls than boys laughed at that joke* is natural, but *#Fewer floors than ceilings laughed...*, *#Fewer children than ceilings laughed...*, and *#Fewer floors than children laughed...*

Here we are concerned with properties of DNPs that are due to their Dets rather than their Ns. As a first case, we consider several *monotonicity generalizations*, including the linguistic problem of characterizing the subject NPs which license negative polarity items, discussed in depth in Ladusaw (this volume). We shall later provide a general definition of monotonicity which will have (15) and (16) as special cases:

(15) A function F from properties to truth values is *increasing* if and only if for all properties A, B if  $F(A) = \mathbf{T}$  and  $A \subseteq B$  then  $F(B) = \mathbf{T}$ . An NP is said to be *increasing* if it always denotes an increasing function. One verifies that an NP X is increasing by checking that it satisfies **Test 1** below (making changes in number agreement where appropriate):

**Test 1** If all As are Bs and X is an A then X is a B

For example *more than six women* is increasing since if all A's are B's and the number of women with property A is more than six then all those women with property A are also ones with property B, so more than six women are B's. By contrast *fewer than six women* is not increasing: Imagine a situation in which, exactly four women are A's, all A's are B's, and there are many B's who are not A's, including as it happens two women. Then *Fewer than six women are A's* is true

but *Fewer than six women are B's* is false, so *fewer than six women* is not increasing. The same situation shows that *exactly four women* is also not increasing.

Whether a DNP is increasing is determined by the choice of Det and not the choice of N. If [D + *women*] is increasing then so is [D + *men*], [D + *students who John knows*], etc. For the most part, syntactically simple (= *lexical*) NPs are increasing. Here is a snapshot of the lexical NPs of English: they include one productive subclass, the Proper Nouns: *John, Mary, ..., Siddartha, Chou en Lai, ...* ('productive' = new members may be added without changing the language significantly). They also include listable sprinklings of (i) personal pronouns - *he/him,...* and their plurals *they/them*; (ii) demonstratives - *this/that* and *these/those*; (iii) possessive pronouns - *his/hers .../theirs*; and (iv) possibly the "indefinite pronouns" *everyone, everybody; someone/body; and no one/body*, though these expressions appear to have meaningful parts. We might also include some uses of Dets, as *all* in *A good time was had by all*, *some* in *Some like it hot*, and *many* and *few* in *Many are called but few are chosen*, though the lexical status of these "NPs" is again doubtful as we are inclined to interpret them as having an understood N *people* to account for their +*human* interpretation (a requirement not imposed by the Dets themselves).

Now one verifies that except for *few* and the "n" words (*no one, nobody*) the lexical NPs above are increasing. Moreover the exceptions, while not increasing, have the dual property of being *decreasing*.

(16) A function F from properties to truth values is *decreasing* iff for all properties A,B if  $A \subseteq B$  and  $F(B) = \mathbf{T}$  then  $F(A) = \mathbf{T}$ . To verify that an NP X is decreasing verify that it satisfies **Test 2**:

**Test 2** If all A's are B's and X is a B then X is a

Clearly if all A's are B's and the number of women who are B's is less than six then the number who are A's must be less than six (if it were six or greater then all those women would be B's, making the number of women who are B's greater than six, contrary to assumption). Thus *fewer than six women* is decreasing. So is *no woman* and so are *no one, nobody* and *few (people)*. Now, calling a function *monotonic* if it is either increasing or decreasing and calling an NP *monotonic* if it always denotes a monotonic function we claim

(17) Lexical NPs are always monotonic, almost always monotonic increasing

(18) Lexical De<sub>1</sub>s always form "continuous" NPs, usually monotonic (increasing)

(17) has already been supported, but we should note that many NPs are not monotonic at all (whence even the weak form of (17) is logically very non-trivial). We have already seen that *exactly four women* is not increasing. But equally it is not decreasing: if all A's are B's and exactly four women are Bs it might be that just two are A's, the other two being B's who are not A's. Thus *exactly four women* is neither increasing nor decreasing, hence not monotonic. Here are some other NPs that are not monotonic:

(19) between five and ten students, about a hundred students, every/no student but John,

every student but not every teacher, both John and Bill but neither Sam nor Mary, most of the students but less than half the teachers, either fewer than five students or else more than a hundred students, more boys than girls, exactly as many boys as girls

Thus the kinds of functions denotable by the NPs in (19) are not available as denotations for lexical NPs in English. In fact K&S show that over a finite universe each GQ is denotable by some NP. So (17) is a strong semantic claim about natural language, one that restricts the hypotheses the language learner need consider in learning the meanings of NPs.

To support (18) observe that of the lexical Dets in **3**, most clearly build increasing NPs: *each, every, all, some, my, the, this, these, several, ... most*. But *no* and *neither* clearly build decreasing NPs. *many* and *few* we put aside for the reasons given earlier. The remaining case is bare numerals, like *two*, which are problematic since their interpretation seems to vary somewhat with the linguistic context. In environments like *Are there two free seats in the front row?*, we interpret *two free seats* as *at least two...*, which is clearly increasing. In contexts like *Two students stopped by while you were out* the speaker seems to be using *two students* to designate two people he could identify, and as far as he himself (but not the addressee) is concerned he could refer to them as *they* or *those two students*. So this usage also seems increasing. But in answer to a question *How many students came to the lecture?* - *Two* the sense is "exactly two (students)", which is non-monotonic. It is perhaps not unreasonable to think of the basic uses of bare numerals as increasing (B&C, K&S), and the non-monotonic uses as ones which draw on additional information from the context.

But even if we take as basic the non-monotonic sense of bare numerals it remains true that the GQs denotable by NPs of the form [Det + N] with Det lexical are a proper subset of the set of denotable GQs. Reason: GQs denotable by NPs of the form *exactly n A's* are expressible as a conjunction of an increasing NP and a decreasing one: *Exactly n A's* denotes the same as *At least n A's and not more than n A's*, and Thysse (1983) has shown that meets of increasing and decreasing functions are just the continuous functions, defined by:

(20) A GQ  $F$  over  $E$  is *continuous* iff  $\forall A, B, C \subseteq E$ , if  $A \subseteq B \subseteq C$  and  $F(A) = F(C) = \mathbf{T}$  then  $F(B) = \mathbf{T}$ .

So monotonic NPs are special cases of continuous ones. But many NPs are not continuous. Typical examples are disjunctions of increasing with decreasing NPs (*either fewer than six students or else more than ten students*) or disjunctions of properly continuous ones (*either exactly two dogs or exactly four cats*). Also NPs like *more male than female students* are not continuous. Thus in analogy with the distinction between lexical vs complex NPs we also see that there are functions denotable by complex Dets which are not denotable by lexical ones, examples being the functions denotable by *more male than female* and *either fewer than ten or else more than a hundred*.

Some further monotonicity generalizations are given by the ways the monotonicity of complex NPs and Dets depends on that of the expressions they are built from:

- (21) a. Conjunctions and disjunctions of increasing (decreasing) NPs are increasing (decreasing). The corresponding claims hold for Dets. Thus *John and every student* is increasing since both conjuncts are. And *both John's and Bill's* builds increasing NPs (*both John's and Bill's cats*) since each conjunct builds increasing NPs.
- b. Negation reverses monotonicity. Thus *not[more than six cats]* is decreasing since the NP *more than six cats* is increasing. And *not more than six*, as in *at least two and not more than six*, builds decreasing NPs since *more than six* builds increasing ones.
- c. The monotonicity value of partitives is determined by the Det preceding *of*. Thus *less than five of the students* is decreasing since *less than five* builds decreasing NPs.
- d. Possessive Dets, e.g. [X's] in [X's N], build increasing NPs if X is increasing, decreasing NPs if X is decreasing, and non-monotonic ones if X is not monotonic. Thus *no student's* builds decreasing NPs (*no student's doctor*) since *no student* is decreasing.

A more surprising monotonicity generalization concerns **negative polarity items (npi's)**. To characterize the set of English expressions judged grammatical by native speakers, we must distinguish (22a) and (23a) from their ungrammatical counterparts (22b) and (23b).

- (22) a. John hasn't ever been to Moscow  
 b. \*John has ever been to Moscow
- (23) a. John didn't see any birds on the walk  
 b. \*John saw any birds on the walk

Npi's such as *ever* and *any* above, do not occur freely; classically (Klima 1964) they must be licensed by a "negative" expression, such as *n't* (= *not*). But observe:

- (24) a. No student here has ever been to Moscow  
 b. \*Some student here has ever been to Moscow
- (25) a. Neither John nor Mary saw any birds on the walk  
 b. \*Either John or Mary saw any birds on the walk
- (26) a. None of John's students has ever been to Moscow  
 b. \*One of John's students has ever been to Moscow

The *a*-expressions here are grammatical, the *b*-ones are not. But the pairs differ with respect to their initial NPs, not the presence vs. absence of *n't*. *The linguistic problem*: define the class of NPs which license the npi's, and state what, if anything, those NPs have in common with *n't/not*.

A syntactic attempt to kill both birds with one stone is to say that just as *n't* is a "reduced" form of *not* so *neither...nor...* is a reduced form of [*not (either...or...)*], *none* a reduction of *not one*, and *no* a reduction of *not a*. The presence of *n-* in the reduced forms is thus explained as a remnant of the original *not*. So on this view the licensing NPs above "really" have a *not* in their representation, and that is what such NPs have in common with *n't*. Moreover NPs built from *not* do license npi's:

- (27) a. Not a single student here has ever been to Moscow  
 b. Not more than five students here have ever been to Moscow

But (Ladusaw, 1983) this solution is insufficiently general: The initial NPs in the *a*- sentences below license npi's; those in the *b*- sentences do not. But neither present reduced forms of *not*.

- (28) a. Fewer than five students here have ever been to Moscow  
 b. \*More than five students here have ever been to Moscow
- a. At most four students here have ever been to Moscow  
 b. \*At least four students here have ever been to Moscow
- a. Less than half the students here have ever been to Moscow  
 b. \*More than half the students here have ever been to Moscow

A better hypothesis, discovered by Ladusaw (1983), building on the earlier work of Fauconnier (1975, 1979), (See also Zwarts, 1981) is given by:

(29) *The Ladusaw-Fauconnier Generalization (LFG)*

Negative polarity items occur within arguments of monotonic decreasing functions but not within arguments of monotonic increasing functions

(The LFG assumes the properly general definition of *decreasing*; see (32)). Clearly the NPs in (24) - (28) which license npi's are decreasing, and those which do not are not. Also, drawing on (21d) and (21c) we see that the LFG yields correct results for (30) and (31), NP types not considered by Ladusaw or Fauconnier.

- (30) No player's agent should ever act without his consent  
 \*Every player's agent should ever act without his consent  
 Neither John's nor Mary's doctor has ever been to Moscow

- (31) None of the teachers and not more than three of the students have ever been to Moscow

To see what property decreasing NPs have in common with negation we must give the properly general definition of *decreasing*.

(32) a. *partial order* is a pair  $(A, \leq)$ , where  $A$  is a set, the *domain* of the partial order, and  $\leq$  is a binary relation on  $A$  which satisfies, for all  $a, b, d \in A$ ,  $a \leq a$  (reflexivity),  $a \leq b \ \& \ b \leq a \Rightarrow a = b$  (antisymmetry) and  $a \leq b \ \& \ b \leq d \Rightarrow a \leq d$  (transitivity).

b. If  $A$  and  $B$  are domains of partial orders and  $F$  is a function from  $A$  into  $B$ ,

i.  $F$  is *increasing* iff for all  $x, y \in A$ , if  $x \leq y$  then  $F(x) \leq F(y)$ .

(That is, if elements  $x, y$  of  $A$  stand in the order relation in  $A$  then their values  $F(x)$  and  $F(y)$  stand in the order relation in  $B$ . So  $F$  preserves the order.)

ii.  $F$  is *decreasing* iff whenever  $x \leq y$  in  $A$  then  $F(y) \leq F(x)$  in  $B$ . So decreasing functions are ones which reverse the order.

Now it is overwhelmingly the case that the sets in which expressions of an arbitrary categor denote are domains of partial orders.  $P_1$ s denote elements of  $P(E)$ , the collection of subsets of  $E$ , and the ordering relation is just the subset relation,  $\subseteq$ . (One verifies that  $\subseteq$  is reflexive, antisymmetric and transitive). As for Sentence denotations, the set  $\{\mathbf{T}, \mathbf{F}\}$  carries the *implication* order, defined by:  $\forall x, y \in \{\mathbf{T}, \mathbf{F}\}$ ,  $x \leq y$  iff an arbitrary formula of the form "if  $P$  then  $Q$ " is true when  $P$  denotes  $x$  and  $Q$  denotes  $y$ . So the relation obtains in just the following cases:  $\mathbf{T} \leq \mathbf{T}$ ,  $\mathbf{F} \leq \mathbf{T}$  and  $\mathbf{F} \leq \mathbf{F}$ . Again one verifies directly that this relation is a partial order relation. And one sees from our earlier definition (15) that increasing NPs are just the ones which denote order preserving maps from the  $P_1$  order into the implication order. And from (16) we see that decreasing NPs are just the ones that reverse the order relations.

But now we can observe that (ignoring tense) *didn't laugh* denotes  $E - \mathbf{LAUGH}$  the set of objects under discussion that are not in the  $\mathbf{LAUGH}$  set. So **not** (**n't**) maps each subset  $A$  of  $E$  to  $E - A$ . And clearly, if  $A \subseteq B$  then  $E - B \subseteq E - A$ , that is, **not**( $B$ )  $\subseteq$  **not**( $A$ ), so **not** is decreasing. (So is classical logical negation which maps  $\mathbf{T}$  to  $\mathbf{F}$  and  $\mathbf{F}$  to  $\mathbf{T}$ ).

In this way then we find a semantic property that "negative" NPs and negation have in common: They all denote decreasing functions. For more extended and refined discussion see Zwarts (1990) and Nam (to appear). We turn now to a second type of generalization.

### Constraints on Determiner Denotations

In the interpretation of Ss of the form  $[[_{NP} \text{Det} + N] + P_1]$  the role of the noun argument is quite different from that of the  $P_1$  argument. It serves to limit the domain of objects we use the predicate to say something about. This simple idea of *domain restriction* is captured in the literature with two independent constraints: *conservativity* and *extension*.

To say that a Det is conservative (CONS) is to say that we can decide whether Det A's are B's if we know which individuals are A's and which of those A's are B's. So for  $B$  and  $C$  possibly quite different predicate denotations, if it happens that the B's who are A's are the same individuals as the C's who are A's then *Det A's are B's* and *Det A's are C's* must have the same

truth value. Formally, we define:

(33) A function  $D$  from  $P_E$  to GQs is *conservative* iff  $\forall A, B, C \subseteq E$

if  $A \cap B = A \cap C$  then  $D(A)(B) = D(A)(C)$

Note that if  $D$  is CONS as per (33) then for all properties  $A, B$  we have that  $D(A)(B) = D(A)(A \cap B)$ , since  $B$  has the same intersection with  $A$  as  $A \cap B$  does. The converse holds as well. For suppose that for all  $X, Y$   $D(X)(Y) = D(X)(X \cap Y)$ . We show that  $D$  satisfies the condition in (33). Let  $A \cap B = A \cap C$ . Then by the condition on  $D$ ,  $D(A)(B) = D(A)(A \cap B) = D(A)(A \cap C)$ , by the assumption,  $= D(A)(C)$ , again by the condition on  $D$ .  $\square$

To check that Det is CONS check that (34a) and (34b) must always have the same truth value (= are logically equivalent), changing singulars to plurals where appropriate.

- (34) a. Det doctor is a vegetarian  
b. Det doctor is both a doctor and a vegetarian

So e.g. *John's* is CONS since *John's doctor is a vegetarian* is logically equivalent to *John's doctor is both a doctor and a vegetarian*.

The apparent triviality of such equivalences suggests, wrongly, that CONS is a very weak condition. K&S show that, for  $|E| = n$ , the number of conservative functions is  $2^3$ , whereas the total number of functions from  $P_E$  to GQs is  $2^4$ . Thus in a situation with just 2 individuals there are  $2^{16} = 65,536$  functions from  $P_E$  to GQs, but only 512 of these are CONS. So conservativity rules out most ways we might associate properties with NP denotations. For example, the function  $G$  given by  $G(A)(B) = 1$  iff  $|A| = |B|$  is not CONS. Nonetheless,

(35) With at most a few exceptions<sup>1</sup> English Dets denote conservative functions

We note finally here that the format in which Conservativity is stated, (33) or its equivalent, does not require that  $D(A)(B)$  be a truth value, it just requires that  $D(A)(B) = D(A)(C)$  (or  $D(A)(A \cap B)$  in the equivalent formulation). Thus it makes sense to ask whether interrogative Dets such as *Which?* are CONS. An affirmative answer would imply that e.g. *Which students are vegetarians?* and *Which students are both students and vegetarians?* are logically the same question, that is, request the same information. And this seems to be the case - a true answer to either of these questions is a true answer to the other. So interrogative *Which?* is CONS.

Moreover (33) is actually just the special case of conservativity for  $De_1$  denotations. The general statement is: if  $D$  is a  $k$ -place Det denotation (that is a function from  $k$ -tuples of properties to GQs) then  $D$  is CONS iff for all  $k$ -tuples  $A$  and all properties  $B, C$  if  $A_i \cap B = A_i \cap C$  then  $D(A)(B) = D(A)(C)$ . In other words if two predicate properties  $B$  and  $C$  have the same intersection with each noun property then the Det takes the same value at the noun properties and the first predicate property as it does at the noun properties and the second predicate property. Usually for ease of reading in what follows we just give definitions for the case of  $De_1$  denotations, assuming the appropriate generalization to  $Det_k$ 's.

A more subtle constraint satisfied by Det denotations is *Extension* (EXT), first noticed by van Benthem (1984). The intuition is that Dets cannot make crucial reference to objects which fail to have the property expressed by their noun arguments. So English could not have a Det *blik* such that *Blik A's are B's* would mean that there are exactly three things that aren't A's. One might have thought that this was covered by Conservativity but in fact it is not. E.g. given E, the function F defined by  $F(A)(B) = \mathbf{T}$  iff  $|E-A| = 3$  is CONS. The problem here is that once E is given a condition on E-A, the non-As in E, can always be expressed as a condition on As, and conservativity allows that we place conditions on the noun arguments A. (See K&W for worked out examples). To test whether crucial reference to non-As is made we must hold A constant and vary the non-As, E-A. So we now we think of a Det denotation D as a functional which chooses for each universe E a function  $D_E$  from  $P_E$  to the generalized quantifiers over E. Then,

(36) D satisfies *Extension*<sup>2</sup> iff  $\forall E, E'$  with  $A, B \subseteq E$  and  $A, B \subseteq E'$ ,  $D_E(A)(B) = D_{E'}(A)(B)$

Thus if D satisfies extension (EXT) then the truth of  $D_E(A)(B)$  does not change if we change the non-As in the universe (and we may continue to write simply  $D(A)(B)$  instead of  $D_E(A)(B)$  since the truth value does not vary with the choice of E). And we claim:

(37) The denotations of natural language Dets always satisfy Extension

We note that CONS and EXT are independent: the functional D which maps each E to the function  $D_E$  which sends A,B to  $\mathbf{T}$  iff  $A = B$  satisfies EXT but  $D_E$  fails CONS in each (non-empty) E. By contrast the functional F which maps E to **EVERY**<sub>E</sub> if E is finite and **SOME**<sub>E</sub> if E is infinite fails to satisfy EXT but in each universe E,  $F_E$  is CONS.

The combined effect of CONS and EXT, namely Domain Restriction, is a kind of "logica topicality" condition. It says in effect that the head noun determines the relevant universe for purposes of the statement we are concerned with. Worth noting here is that mathematical languages such as those used in Elementary Arithmetic, Euclidean Geometry or Set Theory are special purpose in that the range of things we can talk about is fixed in advance (numbers, points and line, sets). But natural languages are general purpose - speakers use them to talk about anything they want, and common nouns in English provide the means to delimit "on line" what speakers talk about and quantify over.

### Subclasses of Dets: Existential There Ss

So given a universe E, whether an arbitrary Det<sub>1</sub> denotation D holds of a pair A,B can only depend on which individuals are A's and which of those are B's. But Dets differ greatly among themselves with regard to how much of this information they use. Below we distinguish several subclasses together with linguistic generalizations based on these classes.

We call a Det denotation D *intersective* (INT) if we can determine the truth of  $D(A)(B)$  just by checking which individuals lie in  $A \cap B$ . So  $D(A)(B)$  depends just on  $A \cap B$ . For example, **MORE THAN TEN** is INT: to know if *More than ten students applauded* we only need to know about individuals in the intersection of **STUDENT** with **APPLAUD**, we need not concern ourselves

with students who didn't applaud. Also **SOME** in the sense of *at least one* is INT. **SOME**(A)(B) = **T** iff  $A \cap B \neq \emptyset$ , so we decide whether *Some A's are B's* just by checking the set of A's who are B's. Since **SOME** corresponds to the existential quantifier in logic we shall sometimes refer to intersective Dets as *generalized existential* Dets.

By contrast *co-intersective* (= CO-INT) Dets just depend on A-B, the complement of  $A \cap B$  relative to A. For example, whether *All but ten A's are B's* is true is decided just by checking the A-B, the set of A's that are not B's, has cardinality 10. Similarly **EVERY**(A)(B) = **T** iff  $A - B = \emptyset$ . (Note that  $A - B = \emptyset$  iff  $A \subseteq B$ ). Since such Dets include **EVERY** and **ALL BUT TEN** they will also be called *generalized universal*. We define:

(38) a. D is *intersective* (INT) iff for all  $A, A', B, B' \subseteq E$ ,

if  $A \cap B = A' \cap B'$  then  $D(A)(B) = D(A')(B')$

b. D is *co-intersective* (CO-INT) iff for all  $A, A', B, B' \subseteq E$ ,

if  $A - B = A' - B'$  then  $D(A)(B) = D(A')(B')$

(38a) for example says that an intersective D cannot tell the difference between arguments A, B and A', B' if they have the same intersection. That is, whether  $D(A)(B)$  is true just depends on the intersection of A with B.

As we shall be primarily concerned with INT and CO-INT below, let us contrast them first with Dets that are neither. *Proportionality* Dets like *most*, *less than half the* and *over ten per cent (of the)* are an important case in point. To decide whether most students read the Times we must be able to compare the students who do with those who don't. Just knowing which students read the Times is insufficient to decide whether most do; and just knowing which students don't is also insufficient. Formally,

(39) a. D is *basic proportional* iff for some  $0 < n < m$

$\forall A, B D(A)(B) = \mathbf{T}$  iff  $|A \cap B| = (n/m) \cdot |A|$  or  $\forall A, B D(A)(B) = \mathbf{T}$  iff  $|A \cap B| > (n/m) \cdot |A|$

b. The *proportionality* Ds are the non-trivial boolean compounds of basic proportional ones.

From (39b) we see that *exactly a third (of the)* is proportional since it denotes the same function as *at least a third and not more than a third*. Equally (39b) covers *co-proportional* Dets like *all but a third* when the noun argument A is finite - the only case where proportions like  $(n/m) \cdot |A|$  are intuitive. In such a case *All but a third of the A's are B's* must have the same value as *Exactly two thirds of the A's are B's*. The non-triviality condition rules out **ALL** and **NO** (expressible for example as *more than half and less than half*).

Thus to decide whether a Det D relates A and B we must know more if D is a proportionality

one than if D is merely INT or CO-INT. In fact B&C show that even if we restrict the universe E to be finite, *more than half* is not definable in first order logic. This claim extends (see K&W and references cited there) to the full class of basic proportional Dets. These results also imply that cardinal comparative De<sub>2S</sub> like **MORE...THAN...** are not definable in first order; if they were we could use them to define **MORE THAN HALF** in first order by: **MORE THAN HALF**(A)(B) = **MORE**(A∩B)**THAN**(A-B)(E). This just says that *more than half the A's are B's* means *more A's who are B's than A's who are not B's exist*. These remarks lay to rest the issue about whether natural language semantics can be represented in first order logic: it can't.

We consider now the (co-)intersective Dets in more detail. Observe first that the INT Dets considered so far satisfy a stronger condition than intersectivity: they are *cardinal* (CARD): the value of D(A)(B) only depends on |A∩B|, the cardinality of A∩B. Dually the CO-INT Dets are *co-cardinal* (CO-CARD); their value just depends on |A-B|. The formal definitions of these notions follow that for INT and CO-INT, replacing everywhere ∩B with |A∩B|, etc. Some examples:

**SOME CARDINAL DETS:** at least/more than/fewer than/exactly ten; between five and ten; not more than ten; either fewer than ten or else more than a hundred; just finitely many

**SOME CO-CARDINAL DETS:** all, almost all, not all, all but ten; all but at most ten; a but finitely many

Note that not all CARD (CO-CARD) Dets are first order definable: *just finitely many* (and its boolean complement *infinitely many*) is not; nor is *all but finitely many*. So the properly proportional Dets do not have a monopoly on being non-first order definable.

Note also that De<sub>2S</sub> like *more...than...* are INT and in fact CARD. To say that a De<sub>2</sub> D is INT is to say that the value of D(A<sub>1</sub>,A<sub>2</sub>)(B) depends just on the intersections A<sub>1</sub>∩B and A<sub>2</sub>∩B. To say that it is CARD just says that it depends on the cardinalities of these intersections. And in fact we can decide the truth of *More A<sub>1</sub>'s than A<sub>2</sub>'s are B's* if we know |A<sub>1</sub>∩B| and |A<sub>2</sub>∩B| so **MORE...THAN...** is CARD and so a fortiori INT. (For further discussion of such De<sub>2S</sub> see Keenan and Moss (1985) and Beghelli (1992, 1993).

Are there (co-)intersective Dets in English that are not (co-)cardinal? There seem to be no syntactically simple cases, but two candidates for somewhat complex ones are *Exceptive Dets* such as *no/every...but John* in NPs like *no student but John/every student but John* and *Cardinal Adjectival Comparative Dets* like *more male than female* in *more male than female students*. In both cases alternative analyses are certainly possible (see section 3). But interpreting them as Dets they clearly fall into the (co-)intersective class:

(40) a. **(NO A BUT JOHN)(B) = T** iff A∩B = {John}

b. **(EVERY A BUT JOHN)(B) = T** iff A-B = {John}

(40a) says that *No student but John laughed* is true iff the set of students who laughed has only John as an element. So the value of **NO...BUT JOHN** at a pair A,B of properties is decided just by checking their intersection, so **NO...BUT JOHN** is intersective. Dually **EVERY...BUT JOHN**(A)(B) just depends on A-B so it is co-intersective. Equally taking adjectives like *male* and *female* as absolute, that is the male A's are just the A's who are male individuals, so  $\mathbf{MALE}(A) = \mathbf{MALE}(E) \cap A$ , the reader may check that *more male than female* defined in (41) is intersective but not cardinal.

(41) **MORE MALE THAN FEMALE**(A)(B) = **T** iff  $|\mathbf{MALE}(A) \cap B| > |\mathbf{FEMALE}(A) \cap B|$

The notions of (co-)intersectivity play an important role in many linguistically revealing generalizations. Here is a first, non-obvious one. We have been interpreting Ss like *Some swans are black* directly as **SOME(SWAN)(BLAC)**. But in standard logic it is represented as  $(\exists x)(\text{Swan}(x) \ \& \ \text{Black}(x))$ , where the quantification is over all the objects in the model, not just the swans. In our variable free notation this would be **SOME(E)(SWAN  $\cap$  BLAC)**. This formulation eliminates the restriction to swans in favor of quantifying over all the objects in the universe, and it preserves logical equivalence with the original by replacing the original predicate property **BLAC** with an appropriate boolean compound of the noun property and the predicate property, **SWAN  $\cap$  BLAC**. Thus *some* does not make essential use of the domain restriction imposed by the noun argument. The same equivalence obtains if we replace *some* by e.g. *exactly two*. *Exactly two swans are black* is logically equivalent to *Exactly two objects are both swans and are black*.

We shall say then that quantifiers like *some* and *exactly two* are *sortally reducible* meaning that we can eliminate the restriction on the domain of quantification compensating by building a new predicate property as some boolean compound (in our examples it was boolean compounds with *and*) of the original noun property and the original predicate property. Formally

(42) For each E, a function D from  $P_E$  to GQs is *sortally reducible* iff there is a two place boolean function h such that for all A,B  $\subseteq E$ ,  $D(A)(B) = D(E)(h(A,B))$ . D is called *inherently sortal* if there is no such h.

**Quer** : Which English Dets are sortally reducible, and which are inherently sortal? One other reducible one comes to mind immediately, based on the standard logic translation *All swans are black*:  $(\forall x)(\text{Swan}(x) \rightarrow \text{Black}(x))$ . This just says that **ALL(SWAN)(BLAC)** = **ALL(E)( $\neg$ SWAN  $\vee$  BLAC)**, where here the boolean compound has been built with complement and disjunction (drawing on the equivalence of  $(P \rightarrow Q)$  and  $(\neg P \vee Q)$ ). Thus like *some*, *all* is sortally reducible. By contrast *most* is not. *Most swans are black* provably has no paraphrase of the form (For most x)(...Swan(x)...Black(x)...) where the expression following (For most x) a formula built from the predicates *Swan* and *Black* (as many times as we like) using any combination of *and*, *or*, and *not*. Now in fact Keenan (1993) provides a complete answer to the query:

(43) A conservative D from  $P_E$  to  $GQ_E$  is sortally reducible iff D is intersective or D is

co-intersective

We turn now to the role of intersectivity in providing an answer to a problem which arises in the context of generative grammar. Consider:

- (44) There wasn't more than one student at the party  
Are there more dogs than cats in the garden?  
There was no one but John in the building at the time  
Weren't there more male than female students at the party?

Such Ss, called Existential There (ET) sentences, are used to affirm, deny or query the existence of objects (e.g. students) with a specified property (e.g. being at the party). NPs like *more than one student* which naturally occur in such Ss will be called *existential NPs*. So the NPs italicized in (45) are not existential, as the Ss are either ungrammatical or assigned an unusual interpretation.

- (45) \*There wasn't *John* at the party  
\*There were *most students* on the lawn  
\*Was there *every student* in the garden?  
\*There wasn't every student but John in the garden  
\*Were there two out of three students in the garden  
\*There weren't John's ten students at the party

*The linguistic problem:* define the set of existential NPs in English. B&C, drawing on Milsark (1977), were the first to propose a semantic solution to this problem, and they located the solution in the nature of the Dets rather than the NPs themselves. In (46) we present a somewhat different solution, one that draws on theirs and on Keenan (1987a). See ter Meulen and Reuland (1987) for extensive discussion of the empirically problematic issues here.

- (46) NPs which occur naturally in ET contexts are (boolean compounds of) ones built from intersective Dets

The data in (44) and (45) support (46). The clause on boolean compounds correctly predicts the acceptability of Ss like *There were about five dogs and more than ten cats in the kennel*. Equally one proves that boolean compounds of intersective Dets are intersective, whence (46) predicts good *There were neither exactly two nor exactly four cats on the mat*.

Thus we have a linguistic property which correlates reasonably well with intersectivity. No corresponding property correlating with co-intersectivity is known. But there is a non-obvious semantic property determined jointly by the INT and CO-INT Dets. Namely, for each universe E,  $CONS_E$  (the set of conservative functions over E) is exactly the functions which are buildable by boolean operations on the intersective and co-intersective functions. Here the negation of a Det, e.g. *not more than ten*, denotes the boolean complement of the Det denotation,  $\neg(\mathbf{MORE\ THAN\ TEN})$ , where in general  $\neg D$  is that function mapping A,B to  $\neg(D(A)(B))$ , where of course  $\neg \mathbf{T} = \mathbf{F}$

and  $\neg\mathbf{F} = \mathbf{T}$ . Similarly  $D_1 \wedge D_2$  is that map sending each  $A, B$  to  $D_1(A)(B) \wedge D_2(A)(B)$ . So *At least two and not more than ten A's are B's* is true iff at least two A's are B's and not more than ten A's are B's. And we claim (Keenan 1993):

(47) For each universe  $E$ ,  $\text{CONS}_E =$  the complete boolean closure of  $\text{INT}_E \cup \text{CO-INT}_E$

In other words, modulo the boolean operations on Det denotations we can represent  $\text{CONS}_E$ , the possible Det denotations over  $E$ , by the generalized existential and generalized universal ones. This is a surprisingly large reduction. Most elements of  $\text{CONS}_E$  are not intersective or co-intersective. E.g. in a universe of just 3 individuals there are more than 130 million conservative functions, only 510 of which are either intersective or co-intersective. The general figure:

(48) For  $|E| = n$ ,  $|\text{CONS}_E| = 2^{3^n}$  and  $|\text{INT}_E \cup \text{CO-INT}_E| = 2^{2^n + 1} - 2$

A last property of intersective Dets concerns the role INT plays in yielding the cardinal Dets from the more general class of "logical" Dets. Alongside of such classical quantifiers as *some*, *every*, and *most* we have been treating as Dets "non-logical" expressions such as *John's* and *more male than female*. But for various purposes, such as mathematical or logical study, we want to distinguish these classes, and we can do so in terms of properties of their denotations. Informally first, the "logical" Dets are ones which cannot distinguish among properties according to which particular individuals have them. So such Dets do not themselves make any contingent (= empirical) claims about how the world is. Thus once we are given the set of cats and the set of black things in a situation the truth of *Some/All/Most cats are black* is determined, but that of *John's cats are black* is not. In this latter case we must still distinguish among the individual cats according as they are John's or not, and which cats John "owns" in a given situation is a contingent property of that situation and may be different in another situation (even with the same universe). (See K&S for more extensive discussion). Formally the "logical" Dets are the isomorphism invariant ones:

(49)  $D$  is *isomorphism invariant* (ISOM) iff  $\forall E$  and all bijections  $\Pi$  with domain  $E$ ,  
 $\Pi(D_E) = D_{\Pi E}$

By  $\Pi E$  is meant  $\{\Pi(x) | x \in E\}$  and similarly for  $\Pi A$  and  $\Pi B$ .  $S_{\Pi}$  is just a way of replacing the objects in  $E$  without omitting any or collapsing any (= mapping different ones to the same one). And by  $\Pi(D_E)$  is meant that map which sends each  $\Pi A, \Pi B$  to whatever  $D_E$  send  $A, B$  to. So (49) says in effect that  $D_E(A)(B) = D_E(\Pi A)(\Pi B)$ , all bijections  $\Pi$  with domain  $E$ . And this just means that the value of  $D(A)(B)$  doesn't change if we systematically replace some individuals by others.

One shows that all (co-)cardinal Dets are ISOM. So are the proportionality Dets like *half of the* (but not *half of John's*), and so are definite Dets like *the ten* (but not *John's ten*) which is neither proportional nor (co-)cardinal. And we observe that the cardinal Dets are just the ISOM intersective Dets.

(50) If  $D$  satisfies Extension<sup>2</sup> then

- a. D is cardinal iff D is isomorphism invariant and intersective, and
- b. D is co-cardinal iff D is isomorphism invariant and co-intersective

### Definite Dets and NPs

The last result we consider here is again based on B&C. We are concerned to define the set of NPs which occur grammatically following the *of* phrase in partitives such as *more than ten of John's cats*, *each of those students*, and *all but two of his ten children*. Linguists usually consider that such NPs have the form [Det<sub>1</sub> of NP], one we generalize to [Det<sub>k</sub> (of NP)<sup>k</sup>] to account for such NPs as *more of John's cats than of Bill's dogs* of the form [Det<sub>2</sub> (of NP)<sup>2</sup>].

Now it turns out that the acceptability of an NP in the *pos of* position in partitives is significantly determined by its choice of Det (which is in part why, earlier, we treated *more than ten of John's*, *each of those*, etc. as complex Det<sub>1</sub>s). Observe:

- (51) a. [*at least two of Det cats*] is acceptable when Det = *the, the six, the six or more, John's (six (or more)), those(six(or more)), John's doctor's (six(or more))*
- b. [*at least two of Det cats*] is not acceptable when Det = *each, no, most, at least/exactly/less than nine, no children's (six)*

To characterize the Dets which build NPs acceptable in plural partitives we need two preliminary observations. First, given  $A \subseteq E$ , we say that a function  $G$  from properties to truth values is *the filter generated by A* iff for all  $B$ ,  $G(B) = \mathbf{T}$  iff  $A \subseteq B$ . **EVERY**(A) is the filter generated by A and **THE SIX**(A) is the filter generated by A if  $|A| = 6$ , otherwise it is  $\mathbf{0}$ , that **GO** which sends each subset B of E to 0. And **(JOHN'S SIX)**(A) is the filter generated by (**which John has**) if in fact John has exactly six A's, otherwise it is  $\mathbf{0}$ .

Secondly, a point we have been ignoring for simplicity of presentation, given a universe E there are typically many acceptable ways of interpreting a non-logical Det such as *John's two*. These interpretations differ according which individual *John* denotes and, for each such individual, which objects that individual stands in the **HAS** relation to. In the general case then we should think of Dets D as functionals associating with each universe E a set  $D_E$  of functions from  $\mathcal{P}_E$  to GQs over E. Our earlier definitions of notions like *cardinal* lift easily<sup>2</sup> to the more general setting. E.g. D is **CARD** iff for each E, each  $F \in D_E$  is **CARD** as defined earlier (viz.,  $F(A)(B) = F(A')(B')$  whenever  $|A \cap B| = |A' \cap B'|$ , all subsets  $A, A', B, B'$  of E). We may now define:

- (52) D is *definite* iff D is non-trivial and for all E, all  $F \in D_E$ , all  $A \subseteq E$ ,  $F(A) = \mathbf{0}$  or  $F(A)$  is the filter generated by some non-empty  $C \subseteq A$ . If C always has at least two elements D is called *definite plural*.

The Dets in (51a) are definite plural and those in (51b) are not. And we propose:

- (53) NPs which occur in plural partitive contexts like [*two of \_\_\_\_\_*] are (conjunctions and disjunctions of) ones built from definite plural Det<sub>1</sub>s.

**Conclusion** These remarks cover several of the major applications of generalized quantifier theory to linguistic analysis. For each generalization we have made we encourage the reader to consult the literature referred to. Of necessity an overview omits much fine grained linguistic analysis in favor of comprehensiveness.

### Footnotes

\* in *The Handbook of Contemporary Semantic Theory* S. Lappin (ed) Blackwell 1996 pp 41-63

1. See for example K&S, Johnsen (1987), Westerståhl (1985) and Herburger (to appear). All putative counterexamples to Conservativity in the literature are ones in which a sentence of the form *Det A's are B's* is interpreted as  $D(B)(A)$ , where  $D$  is conservative. So the problem is not that the Det fails to be conservative, rather it lies with matching the Noun and Predicate properties with the arguments of the Det denotation. Thus Westerståhl points out that Ss with *many* as *Many Scandinavians have won the Nobel Prize* admit of an interpretation in which  $i$  means that many Nobel Prize winners have been Scandinavian. Herburger points out that this reversal of the domain restricting and predicate properties is not allowed by all Dets. E.g. *Most Scandinavians have won the Nobel Prize* does not allow such reversal. And the whiff of generality is in the air: Can we find a property of Dets which allows us to predict whether they allow the reversal of the Noun and Predicate properties

2. See K&W for the appropriate generalization of "satisfies Extension" to the case of "non-logical" Dets, in which case a Det  $D$  associates with each non-empty universe  $E$  a set of functions from  $P_E$  to  $GQ_E$ .

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