Pregroup grammars and Chomsky's earliest examples.[1)]

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Pregroups are partially ordered monoids in which each element has two "adjoints". Pregroup grammars provide a computational approach to natural languages by assigning to each word in the mental dictionary a *type*, namely an element of the pregroup freely generated by a partially ordered set of basic types. In this expository article, the attempt is made to introduce linguists to a pregroup grammar of English by looking at Chomsky's earliest examples.

1. Introduction.

Pregroup grammar is a recent development of categorial grammar, which asserts that grammatical calculations are to be performed on the types which have been assigned to the words in the mental dictionary, but which live in an algebraic or logical system. I believe that this computational approach is compatible with the profound insights gained by Chomsky and his school in the last half century.

Ever since his pioneering Syntactic Structures [C1957] revolutionized the science of linguistics, Chomsky's theories have undergone a number of mutations, from generative transformational grammar via Government and Binding to the more recent Minimalist Program. It is not clear to me how many of the theories underlying the intermediate stages are still relevant, hence I will concentrate here on some of the earliest examples, which were introduced in [C1957] to challenge the then prevailing linguistic orthodoxy and which still offer a challenge to anyone trying to construct a grammar of English.

2. Pregroup grammar.

Without going too deeply into the mathematical background, let me describe briefly what a *pregroup grammar* of a natural language, say of English, looks like. We assume that each word in the mental dictionary has been assigned one or more types. A *type* is defined to be a string of *simple types*, each of the form

$$\cdots, x^{\ell\ell}, x^{\ell}, x, x^{r}, x^{rr}, \cdots$$

where x is a basic type. *Basic types* are assumed to be elements of a partially ordered set, the partial order being denoted by an arrow. There are only two rules of computation:

$$x^{\ell}x \to 1 \to xx^{\ell}, \qquad xx^{r} \to 1 \to x^{r}x, \tag{2.1}$$

when x is any simple type. We may think of the arrow in (2.1) as extending the partial order from basic types to types. In a terminology inspired by category theory, we call x^{ℓ} the *left adjoint* and x^{r} the *right adjoint* of x. Readers interested in the mathematical background are invited to consult Section 17.

Let me make just one historical remark. A similar grammar has been foreshadowed by Z. Harris [H 1966, 1968], but without iterated adjoints. In fact, double adjoints have so far only

been shown to serve a useful purpose in modern European languages to explain Chomskyan traces and to describe clitics in Romance languages. As to triple adjoints, the jury is still out. Harris used *contractions* $x^\ell x \to 1$ and $xx^r \to 1$, having no need for *expansions* $1 \to xx^\ell$ and $1 \to x^r x$. Why expansions are not needed for most linguistic purposes will be explained in Section 17. However, they are required for theoretical reasons, for example, to justify extending the partial order and the adjoint operations to simple types and strings of simple types. One may prove the following:

(2.2) if $x \to y$ then $y^\ell \to x^\ell$ and $y^r \to x^r$,
hence $x^{\ell\ell} \to y^{\ell\ell}$ and $x^{rr} \to y^{rr}$;

(2.3) $x^{\ell r} = x = x^{r\ell}$;

(2.4) $(xy)^\ell = y^\ell x^\ell, \qquad (xy)^r = y^r x^r.$

3. Some simple sentences.

Let me begin by aiming at the much quoted sentence:

(3.1) *Colourless green ideas sleep furiously*,

taken from [C 1957]. Any reader, unhappy with the semantics of (3.1), may substitute

(3.2) *pointless new ideas die rapidly.*

First take a look at the simpler

(3.3)
$$\begin{matrix} & & \textit{ideas sleep} \\ \mathbf{p} & (\pi_2^r & \mathbf{s}_1) \end{matrix} \quad \to \ \mathbf{s}_1.$$

Here the following basic types make their first appearance:

$\mathbf{p}$ = (type of) plural noun,

π_2 = plural subject pronoun (including the second person singular, since the old *thou* has disappeared),

$\mathbf{s}_1$ = declarative sentence in the present tense.

We postulate

$$\mathbf{p} \to \pi_2$$

in the partially ordered set of basic types. The underlink in (3.3) indicates the generalized contraction

$$\mathbf{p}\pi_2^r \to \pi_2\pi_2^r \to 1.$$

We look at some variations of (3.3):

(3.4)
$$\begin{matrix} & & & & \textit{ideas can sleep} \\ \mathbf{p} & (\pi_2^r & \mathbf{s}_1 & \mathbf{j}^\ell) & \mathbf{i} \end{matrix} \quad \to \ \mathbf{s}_1$$

(3.5)
$$\begin{matrix} & & & & & & & \textit{ideas can have slept} \\ \mathbf{p} & (\pi_2^r & \mathbf{s}_1 & \mathbf{j}^\ell) & (\mathbf{j} & \mathbf{p}_2^\ell) & \mathbf{p}_2 \end{matrix} \quad \to \ \mathbf{s}_1$$

(3.6)
$$\begin{array}{c} \textit{ideas do sleep} \\ \mathbf{p}\ \underbrace{(\pi_2^r \mathbf{s}_1 \mathbf{i}^\ell)}\ \underbrace{\mathbf{i}} \end{array} \quad \to\ \mathbf{s}_1$$

but not

(3.7)
$$\begin{array}{c} {}^*\textit{ideas do have slept} \\ \mathbf{p}\ \underbrace{(\pi_2^r \mathbf{s}_1 \mathbf{i}^\ell)}\ \underbrace{(\mathbf{j}}\ \underbrace{\mathbf{p}_2^\ell)\ \mathbf{p}_2} \end{array} \quad \not\to\ \mathbf{s}_1$$

Here we have employed the following additional basic types:

$\mathbf{j}$ = intransitive infinitive,

$\mathbf{i}$ = infinitive of intransitive verbs,

$\mathbf{p}_2$ = past participle of intransitive verbs.

We postulate

$$\mathbf{i} \to \mathbf{j} \text{ (but } \mathbf{j} \not\to \mathbf{i}).$$

The distinction between $\mathbf{i}$ and $\mathbf{j}$ serves to explain the different behaviour of the modal verb *can* and the emphatic auxiliary verb *do*. Note that

$$\mathbf{j}^\ell \mathbf{i} \to \mathbf{j}^\ell \mathbf{j} \to 1; \text{ but } \mathbf{i}^\ell \mathbf{j} \not\to 1,$$

since $\mathbf{i}^\ell \mathbf{j} \to 1$ would imply

$$\mathbf{j} = 1\mathbf{j} \to \mathbf{i}\mathbf{i}^\ell \mathbf{j} \to \mathbf{i}1 = \mathbf{i}.$$

4. Adverbs.

Next, consider the simple sentence

(4.1)
$$\begin{array}{c} \textit{ideas can sleep furiously} \\ \mathbf{p}\ \underbrace{(\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell)}\underbrace{[\mathbf{i}\ \underbrace{(\mathbf{i}^r \mathbf{i})}} \end{array} \quad \to\ \mathbf{s}_1$$

Here we have assigned the type $\mathbf{i}^r\mathbf{i}$ to the adverb *furiously*, so that *sleep furiously* has type

$$\mathbf{i}\mathbf{i}^r\mathbf{i} \to \mathbf{i}1 = \mathbf{i}.$$

The left square bracket [before the occurrence of $\mathbf{i}$ in (4.1) serves as a kind of punctuation mark, to indicate that the tempting contraction $\mathbf{j}^\ell\mathbf{i} \to \mathbf{j}^\ell\mathbf{j} \to 1$ is to be postponed.

Actually, we may contrast the *surface structure* (apologies to Chomsky)

$$\mathbf{p}(\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell)\mathbf{i}(\mathbf{i}^r\mathbf{i})$$

with the *deep structure*

$$[\mathbf{p}\pi_2^r]\mathbf{s}_1[\mathbf{j}^\ell[\mathbf{i}\mathbf{i}^r]\mathbf{i}].$$

Most of these square brackets have been replaced by the underlinks in (4.1), and only the single left bracket [between $\mathbf{j}^\ell$ and $\mathbf{i}$ has been retained to indicate that the contraction is to be postponed.

5. Conjugation.

Now let us leave out the modal verb *can* in (4.1) and consider

$$\begin{array}{c} \textit{ideas sleep furiously} \\ \mathbf{p}\ \underbrace{(\pi_2^r \mathbf{s}_1)}\ (\mathbf{i}^r\mathbf{i}) \end{array} \qquad \stackrel{?}{\to} \mathbf{s}_1 \tag{5.1}$$

It seems convenient to analyze the finite verb form *sleep* occurring here as being the result of applying an *inflector* C_{12} of type $\pi_2^r\mathbf{s}_1\mathbf{j}^\ell$ to the infinitive of *sleep* of type $\mathbf{i}$. Then (5.1) may be re-analyzed as follows:

$$\begin{array}{c} \textit{ideas } C_{12} \textit{ sleep furiously} \\ \mathbf{p}\ \underbrace{(\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell)}\ \underbrace{[\mathbf{i}\ \underbrace{(\mathbf{i}^r\mathbf{i})}]} \end{array} \qquad \to\ \mathbf{s}_1. \tag{5.2}$$

More generally, we may put

$$\begin{array}{l} C_{jk} = j - \text{th tense},\ k - \text{th person, so that} \\ C_{jk}\ \textit{sleep}\ \to \begin{pmatrix} \textit{sleep sleep sleeps} \\ \textit{slept slept slept} \end{pmatrix} \end{array}$$

with $j = 1, 2$ and $k = 1, 2, 3$.

Note that, in English, the finite verb forms may be displayed in a matrix with $2 \times 3 = 6$ entries, ignoring the almost obsolete subjunctive, whereas literary French requires $7 \times 6 = 42$ finite verb forms, and Latin even $3 \times 5 \times 6 = 90$. Anyway, for the present purpose, we write

$$\begin{array}{rcl} C_{12}\ \textit{sleep} & \to & \textit{sleep} \\ (\pi_2^r \mathbf{s}_1 \underbrace{\mathbf{j}^\ell})\mathbf{i} & \to & \pi_2^r \mathbf{s}_1 \end{array} \tag{5.3}$$

This is opposed to

$$\begin{array}{rcl} C_{13}\ \textit{sleep} & \to & \textit{sleeps} \\ (\pi_3^r \mathbf{s}_1 \underbrace{\mathbf{j}^\ell})\mathbf{i} & \to & \pi_3^r \mathbf{s}_1 \end{array} \tag{5.4}$$

and

$$\begin{array}{rcl} C_{2k}\ \textit{sleep} & \to & \textit{slept} \\ (\pi^r \mathbf{s}_2 \underbrace{\mathbf{j}^\ell})\mathbf{i} & \to & \pi^r \mathbf{s}_2 \end{array} \tag{5.5}$$

where

$\mathbf{s}_2$ = declarative sentence in the past tense,

π = subject pronoun when the person does not matter,

and we postulate

$$\pi_k \to \pi \qquad (k = 1, 2, 3).$$

In (5.3) to (5.5), the upper arrow belongs to generative morphology and the lower arrow to pregroup grammar.

6. Nouns and adjectives.

The plural *ideas* (of type $\mathbf{p}$) is the plural of the count noun *idea*, say of type $\mathbf{c}$. It might be analyzed further by assigning the type $\mathbf{c}^r\mathbf{p}$ to the morpheme $+\mathbf{s}$. However, I prefer introducing an *inflector* Plur of type $\mathbf{pc}^\ell$, so that

$$\begin{array}{c} \text{Plur } idea \ \rightarrow \ ideas \\ \underbrace{(\mathbf{pc}^\ell)}\ \mathbf{c} \ \rightarrow \ \mathbf{p} \end{array} \tag{6.1}$$

Concentrating on plurals has the advantage that they don't require determiners. (The same would be true for mass nouns, say of type $\mathbf{m}$, which don't have plurals.)

Adjectives can play two different rôles as predicates and as attributes and must be given different types accordingly, say $\mathbf{a}$ and xx^ℓ, where $x = \mathbf{p}, \mathbf{c}$ or $\mathbf{m}$, in particular $x = \mathbf{p}$ when modifying a plural. Thus

$$\begin{array}{c} ideas\ are\ green \\ \underbrace{\mathbf{p}}\ \underbrace{(\pi_2^r\mathbf{s}_1\mathbf{a}^\ell)}\ \underbrace{\mathbf{a}} \quad \rightarrow \ \mathbf{s}_1 \end{array} \tag{6.2}$$

$$\begin{array}{c} green\ ideas\ sleep \\ \underbrace{(\mathbf{pp}^\ell)\ \underbrace{\mathbf{p}}\ (\pi_2^r\mathbf{s}_1)} \quad \rightarrow \ \mathbf{s}_1 \end{array} \tag{6.3}$$

It is easy to replace the predicate *green* by

$$\begin{array}{c} very\ green \\ \underbrace{(\mathbf{aa}^\ell)}\ \underbrace{\mathbf{a}} \quad \rightarrow \ \mathbf{a} \end{array} \tag{6.4}$$

but it is less obvious how *very* can modify the attribute *green*. One is tempted to handle this by postulating $\mathbf{a} \rightarrow \mathbf{pp}^\ell$, so that *very green* has type

$$(\mathbf{aa}^\ell)\mathbf{a} = \mathbf{a}(\mathbf{a}^\ell\mathbf{a}) \rightarrow \mathbf{a} \rightarrow \mathbf{pp}^\ell$$

as befits an attribute. But adopting such a postulate would go counter to our strategy of allowing only postulates of the form $x \rightarrow y$, where x and y are basic types. This is in the interest of keeping the grammar context-free [B2001]. Instead, I propose to assign to *green*, as well as to all adjectives of type $\mathbf{a}$, the additional type

$$\mathbf{a}(\mathbf{a}^r\mathbf{pp}^\ell) \rightarrow \ \mathbf{pp}^\ell,$$

so that

$$\begin{array}{c} very\ green \\ \underbrace{(\mathbf{aa}^\ell)\underbrace{(\mathbf{aa}^r\mathbf{pp}^\ell)}} \ \rightarrow \ \mathbf{pp}^\ell \end{array} \tag{6.5}$$

when used as an attribute.

Instead of listing the additional type separately for each adjective in the dictionary, we may adopt the following *metarule* (apologies to Gazdar):

Metarule 6.6. All adjectives of type **a** may have an invisible ending of type $\mathbf{a}^r x x^\ell$, where $x = \mathbf{p}, \mathbf{c}$ or $\mathbf{m}$.

Such an invisible ending may in fact be justified on historical grounds. In German, a language closely related to English, every attributive adjective must carry a *visible* ending

$$+e,\ +em,\ +en,\ +er,\ +es,$$

which encodes gender, number, case and definiteness. Of course, the invisible ending in English carries no such information; the same adjectival form can modify plurals, count nouns and mass nouns, whatever the case or gender.

7. Chomsky's first example.

At last, we are in a position to analyze (3.1) as follows:

(7.1)
$$\begin{array}{l} \textit{colourless green ideas sleep furiously} \\ (\underline{\mathbf{a}\mathbf{a}^r}\,\mathbf{p}\,\mathbf{p}^\ell)(\underline{\mathbf{a}\mathbf{a}^r}\,\mathbf{p}\,\mathbf{p}^\ell)\ \mathbf{p}\ (\pi_2^r\,\mathbf{s}_1\,\mathbf{j}^\ell[\mathbf{i}])(\mathbf{i}^r\,\mathbf{i}) \quad \rightarrow\ \mathbf{s}_1 \end{array}$$

with the left square bracket to remind us that the sentence does not end after *sleep*.

In the interest of morphology, we might have analyzed *furiously* as

(7.2)
$$\begin{array}{l} \textit{furious}\ +\ \textit{ly}\ \rightarrow\ \textit{furiously} \\ \mathbf{a}\ (\mathbf{a}^r\mathbf{i}^r\mathbf{i})\ \rightarrow\ \mathbf{i}^r\mathbf{i} \end{array}$$

which would have allowed us to justify applying the modifier *very* to yield

(7.3)
$$\begin{array}{l} \textit{very furiously} \\ (\mathbf{a}\mathbf{a}^\ell)\ (\mathbf{a}\mathbf{a}^r\mathbf{i}^r\mathbf{i}) \quad \rightarrow\ \mathbf{i}^r\mathbf{i}. \end{array}$$

Unfortunately, this does not explain why there is no corresponding adverb **greenly*. We might also have been tempted to analyze

$$\begin{array}{l} \textit{colour}\ +\ \textit{less}\ \rightarrow\ \textit{colourless} \\ \mathbf{c}\qquad (\mathbf{c}^r\mathbf{a})\ \rightarrow\ \mathbf{a} \end{array}$$

but this would have wrongly predicted

$$\begin{array}{l} {}^*\textit{green colour}\ +\ \textit{less} \\ (\mathbf{c}\mathbf{c}^\ell)\quad \mathbf{c}\qquad (\mathbf{c}^r\mathbf{a})\ \rightarrow\ \mathbf{a} \end{array}$$

So, it may be wiser to avoid this tempting excursion into morphology for the time being.

8. Participles.

Still in Syntactic Structures [C1957], Chomsky contrasts the grammatical

(8.1*a*) *the book seems interesting*

with the ungrammatical

(8.1*b*) **the child seems sleeping.*

He points out, of course, that *interesting* is an adjective and that *sleeping* is not. What is surprising about this example is this: usually present participles of transitive verbs do not behave like adjectives, but those of intransitive verbs do, the converse being true about past participles.

One way to approach participles is to replace the type $\mathbf{i}$ by $\mathbf{p}_j$, where the subscript $j = 1$ for the present tense and $j = 2$ for the past. Thus we have, for example,

(8.2) $$sleeping : \mathbf{p}_1, \quad dismissed : \mathbf{p}_2\mathbf{o}^\ell$$

where

$$\mathbf{o} = \text{ (type of) direct object.}$$

We then obtain

(8.3) *ideas are sleeping*

$$\underbrace{\mathbf{p}}\ \underbrace{(\pi_2^r \mathbf{s}_1 \mathbf{p}_1^\ell)}\ \underbrace{\mathbf{p}_1} \quad \rightarrow\ \mathbf{s}_1$$

and

(8.4*a*) *people are dismissing them*

$$\underbrace{\mathbf{p}}\ \underbrace{(\pi_2^r \mathbf{s}_1 \mathbf{p}_1^\ell)}\ \underbrace{(\mathbf{p}_1 \mathbf{o}^\ell)}\ \underbrace{\mathbf{o}}$$

but not

(8.4*b*) **people are dismissing*

Here *sleeping* looks like a predicative adjective and *dismissing* does not.

On the other hand, one may say

(8.5*a*) *ideas are dismissed*

but not

(8.5*b*) **ideas are slept.*

So, *dismissed* looks like a predicative adjective and *slept* does not. Moreover, *sleeping* and *dismissed* can also be used attributively, as in

(8.6*a*) *sleeping dogs*, *dismissed ideas*

yet, they are not adjectives, as we cannot say

(8.6*b*) **very sleeping*, **very dismissed.*

Present participles of intransitive verbs, with type $\mathbf{p}_1$, and past participles of transitive verbs, with type $\mathbf{p}_2\mathbf{o}^\ell$, resemble adjectives inasmuch as they may occur as complements of the copula *be*, just as do adjectives, with type $\mathbf{a}$. Moreover, both may be used attributively with invisible endings of type

$$y^r x x^\ell \ (x = \mathbf{p}, \mathbf{c} \text{ or } \mathbf{m};\ y = \mathbf{a}, \mathbf{p}_1 \text{ or } \mathbf{p}_2\mathbf{o}^\ell).$$

Note that

$$(\mathbf{p}_2\mathbf{o}^\ell)^r = \mathbf{o}^{\ell r}\mathbf{p}_2^r = \mathbf{o}\mathbf{p}_2^\ell$$

according to (2.4) and (2.3). Consider, for example,

(8.7) *let sleeping dogs lie*; *barking dogs don't bite.*

Corresponding endings in German are visible.

Yet, *interesting* is a genuine adjective, and so is *interested* (though this is not discussed in [C1957]). Thus, we have

(8.8*a*) *ideas are very interesting*

$$\underbrace{\mathbf{p}}\ \underbrace{(\pi_2^r}\mathbf{s}_1\underbrace{\mathbf{a}^\ell)}\underbrace{(\mathbf{a}\mathbf{a}^r)}\ \underbrace{\mathbf{a}} \qquad \rightarrow\ \mathbf{s}_1$$

(8.8*b*) *people are very interested*

$$\underbrace{\mathbf{p}}\ \underbrace{(\pi_2^r}\mathbf{s}_1\underbrace{\mathbf{a}^\ell)}\ \underbrace{(\mathbf{a}\mathbf{a}^\ell)}\ \underbrace{\mathbf{a}} \qquad \rightarrow\ \mathbf{s}_1$$

Both *interesting* and *interested* are listed in the paperback Oxford English Dictionary as adjectives.

Chomsky [C1957] takes (8.1a,b) as an opportunity to say "such examples suggest that any search for a semantically based definition of "grammaticalness" will be futile. Conceivably, the next section will challenge this statement.

9. Adjectival participles.

Are there any other verbs that behave like *interest* in allowing both participles to serve as adjectives? Searching the back of my own mind, I am able to come up with the following partial list:

(9.1) *amuse*, *annoy*, *charm*, *convince*, *depress*,
disappoint, *discourage*, *distress*, *disturb*, *excite*,
fascinate, *frighten*, *intimidate*, *intoxicate*,
please, *satisfy*, *surprise*, $\cdots$

Buried in my subconscious is the knowledge that present and past participles of all these verbs are adjectives. Many, but not all of them, are so listed in the Oxford Dictionary. It seems implausible that all these verbs are listed separately in people's mental dictionary as having adjectival participles. More likely, there is some criterion they all satisfy. They are, of course, all transitive verbs requiring an animate direct object; but, more than that, they all describe *causation of an emotional or mental state.* For example,

(9.2) $X\ frightens\ Y \quad \Leftrightarrow \quad X\ causes\ Y\ to\ be\ afraid.$

This criterion is surely a semantic one.[2)] Evidently, our criterion does not tell the whole story, e.g. it does not cover *challenge* and *touch.* The reader will easily come up with a number of verbs for which only one of the two participles is a genuine adjective, e.g. *forbid, heat, illuminate, promise,.....*

Chomsky seems to concede that some semantics should be invoked in his example. He contrasts

(9.2a) $^*John\ frightens\ sincerity$

with

(9.2b) $sincerity\ frightens\ John.$

He has recourse to "degrees of grammaticalness" and suggests that (9.2a) is less grammatical than (9.2b).

A reader might easily deceive herself by thinking that (9.2a) is ungrammatical because *sincerity cannot be frightened*. Yet, in saying this, she has inadvertently employed the passive of the supposedly ungrammatical (9.2a).

10. Compound tenses.

Next, let us look at examples (41) and (42) in [C1957] as viewed in a pregroup grammar. I now find it convenient to introduce two intermediate types between **i** and **j**:

$$\mathbf{i} \ \rightarrow\ \mathbf{i}' \ \rightarrow\ \mathbf{j}' \ \rightarrow\ \mathbf{j}.$$

I will adopt the inflectors

$$\mathrm{Part}_1 : \mathbf{p}_1\mathbf{i}'^{\ell}, \qquad \mathrm{Part}_2 : \mathbf{p}_2\mathbf{j}'^{\ell}$$

for present and past participles, and I will assign the following types to the auxiliary verbs for introducing the *progressive* tense, the *perfect* tense and the *passive* voice respectively:

$$be_{\mathrm{prog}} : \mathbf{j}'\mathbf{p}_1^{\ell},\ have_{\mathrm{part}} : \mathbf{j}\mathbf{p}_2^{\ell},\ be_{\mathrm{pass}} : \mathbf{i}'\mathbf{o}^{\ell\ell}\mathbf{p}_2^{\ell}.$$

These are illustrated by the following examples:

(10.1a)
$$\underset{\textit{they arrive}}{\underbrace{\pi_2(\pi_2^r\mathbf{s}_1\mathbf{j}^{\ell}\mathbf{i})}} \ \rightarrow\ \mathbf{s}_1$$

(10.1b)
$$\underset{\textit{they will arrive}}{\underbrace{\pi_2\ (\pi^r\mathbf{s}_1\mathbf{j}^{\ell})\ \mathbf{i}}} \ \rightarrow\ \mathbf{s}_1$$

(10.1c)
$$\underset{\textit{they have arrived}}{\underbrace{\pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{j}^{\ell}\mathbf{j}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{j}'^{\ell}\mathbf{i})}} \ \rightarrow\ \mathbf{s}_1$$

(10.1d)
$$\underset{\textit{they are arriving}}{\underbrace{\pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{j}^{\ell}\mathbf{j}'\mathbf{p}_1^{\ell})(\mathbf{p}_1\mathbf{j}'^{\ell}\mathbf{i})}} \ \rightarrow\ \mathbf{s}_1$$

(10.1e)
$$\underset{\textit{they do arrive}}{\underbrace{\pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{i}^{\ell})\ \mathbf{i}}} \ \rightarrow\ \mathbf{s}_1$$

(10.1*f*)
$$\begin{array}{c} \textit{they are seen} - \\ \pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell \mathbf{i}' \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i} \mathbf{o}^\ell) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

Note that the modal auxiliaries like *will* and the emphatic *do* have no infinitive in English and that the passive can also be formed with

$$\textit{get}:\ \mathbf{i}\mathbf{o}^{\ell\ell}\mathbf{p}_2^\ell$$

The dash in (10.1f) indicates a Chomskyan *trace*, although this concept had not yet been introduced in [C1957]. The type $\mathbf{o}^{\ell\ell}$ will ultimately be refined in Section 14.

If the sentences (10.1a to e) are to be followed by an adverb such as *today* of type $\mathbf{i}^r\mathbf{i}$, the last occurrence of $\mathbf{i}$ should be blocked by a left bracket, e.g.

(10.2*a*)
$$\begin{array}{c} \textit{they arrive today} \\ \pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell [\mathbf{i})(\mathbf{i}^r \mathbf{i}) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

(10.2*c*)
$$\begin{array}{c} \textit{they have arrived today} \\ \pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell [\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i})(\mathbf{i}^r \mathbf{i}) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

Similarly, (10.2f) may be handled by placing a left square bracket before $\mathbf{i}'$; but here we require an additional type $\mathbf{j}^r\mathbf{j}$ for *today*:

(10.2*f*)
$$\begin{array}{c} \textit{they are seen today} \\ \pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell [\mathbf{i}' \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i} \mathbf{o}^\ell)(\mathbf{j}^r \mathbf{j}) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

11. Compounded compound tenses.

Our type assignment also permits some multiply compound tenses. Varying person and tense, we obtain

(11.1*a*)
$$\begin{array}{c} \textit{I have been arriving} \\ \pi_1\ (\pi_1^r\, \mathbf{s}_1\, \mathbf{j}^\ell \mathbf{j}\, \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{j}'\, \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{i}) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

(11.1*b*)
$$\begin{array}{c} \textit{you will have arrived} \\ \pi_2\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}^\ell \mathbf{i}) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

(11.1*c*)
$$\begin{array}{c} \textit{she will be arriving} \\ \pi_2\ (\pi^r\, \mathbf{s}_1\, \mathbf{j}^\ell)(\mathbf{j}'\, \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{i}) \ \rightarrow\ \mathbf{s}_1 \end{array}$$

(11.1*d*)
$$\begin{array}{c} \textit{we would have been arriving} \\ \pi_2\ (\pi^r \mathbf{s}_2 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{j}' \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{i}) \ \rightarrow\ \mathbf{s}_2 \end{array}$$

(11.1*e*)
$$\underset{\text{you would be seen} -}{\pi_2\ (\pi^r \mathbf{s}_2 \mathbf{j}^\ell)(\mathbf{i}^\ell \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i}\mathbf{o}^\ell)} \ \rightarrow\ \mathbf{s}_2$$

(11.1*f*)
$$\underset{\text{they would have been being seen} -}{\pi_2\ (\pi^r \mathbf{s}_2 \mathbf{j}^\ell)(\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{j}' \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{i}' \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i}\mathbf{o}^\ell)} \ \rightarrow\ \mathbf{s}_2$$

although (11.1f) may only be marginally acceptable.

However, our type assignment will not allow

(11.2*a*)
$$\underset{^*\text{they are having arrived}}{\pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell \mathbf{j}' \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{j}' \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i})} \ \not\rightarrow \mathbf{s}_1$$

as long as $\mathbf{j}' \not\rightarrow \mathbf{i}'$,

(11.2*b*)
$$\underset{^*\text{they have had arrived}}{\pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell \mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{j}\ \mathbf{p}_2^\ell)(\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i})} \ \not\rightarrow \mathbf{s}_1$$

as long as $\mathbf{j} \not\rightarrow \mathbf{j}'$.

(11.2*c*)
$$\underset{^*\text{they are being arriving}}{\pi_2\ (\pi_2^r \mathbf{s}_1 \mathbf{j}^\ell \mathbf{j}' \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{j}' \mathbf{p}_1^\ell)(\mathbf{p}_1 \mathbf{i}'^\ell \mathbf{i})} \ \not\rightarrow \mathbf{s}_1$$

as long as $\mathbf{j}' \not\rightarrow \mathbf{i}'$.

12. Yes-or-no questions.

Chomsky [1957] accounts for yes-or-no questions with the help of transformations. We achieve the same result by assigning new types to modal and auxiliary verbs, as in the following examples, where $\mathbf{q}_i$ = question in the i-th tense.

(12.1*a*)
$$\underset{\text{will they arrive ?}}{(\mathbf{q}_1 \mathbf{j}^\ell \pi^\ell)\pi_2\ \mathbf{i}} \ \rightarrow\ \mathbf{q}_1$$

(12.1*b*)
$$\underset{\text{have they arrived ?}}{(\mathbf{q}_1 \mathbf{p}_2^\ell \pi_2^\ell)\ \pi_2\ (\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i})} \ \rightarrow\ \mathbf{q}_1$$

(12.1*c*)
$$\underset{\text{are they arriving ?}}{(\mathbf{q}_1 \mathbf{p}_1^\ell \pi_2^\ell)\ \pi_2\ (\mathbf{p}_1 \mathbf{j}'^\ell \mathbf{i})} \ \rightarrow\ \mathbf{q}_1$$

(12.1*d*)
$$\underset{\text{are they seen} - ?}{(\mathbf{q}_1 \mathbf{o}^{\ell\ell} \mathbf{p}_2^\ell \pi_2^\ell)\ \pi_2\ (\mathbf{p}_2 \mathbf{j}'^\ell \mathbf{i}\mathbf{o}^\ell)} \ \rightarrow\ \mathbf{q}_1$$

(12.1*e*)
$$\begin{array}{c} \textit{do they arrive ?} \\ (\mathbf{q}_1\mathbf{i}^\ell\pi_2^\ell)\ \pi_2\ \mathbf{i}\ \rightarrow\ \mathbf{q}_1 \end{array}$$

We summarize the new types in the

Metarule 12.2. If the finite form of a *modal* or *auxiliary*[3)] verb has the type $\pi_k^r\mathbf{s}_j x^\ell$ ($x = \mathbf{j}, \mathbf{p}_2, \mathbf{p}_1, \mathbf{p}_2\mathbf{o}^\ell, \mathbf{i}, \mathbf{a}$) in a statement, then it has type $\mathbf{q}_j x^\ell \pi_k^\ell$ in a question. (Similarly with π_k replaced by π.)

Note that, in German, a similar metarule will apply to all verbs; but, in modern English, the following is forbidden:

(12.3) $$^*\textit{arrive they ?}$$

However, in Section 14, we will admit (12.3) as a *pseudo-sentence*, which will serve to analyze such sentences as (14.3) below.

13. Negation.

The easiest way to negate a sentence is to insert the word *not* of type xx^ℓ, where $x = \mathbf{i}, \mathbf{j}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{a}, \cdots$

(13.1*a*)
$$\begin{array}{c} \textit{they will not arrive} \\ \pi_2\ (\pi^r\mathbf{s}_1\mathbf{j}^\ell)(\mathbf{j}\ \mathbf{j}^\ell)\ \mathbf{i} \quad \rightarrow \mathbf{s}_1 \end{array}$$

(13.1*b*)
$$\begin{array}{c} \textit{they have not arrived} \\ \pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{j}^\ell\mathbf{j}\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{j}'^\ell\mathbf{i}) \quad \rightarrow \mathbf{s}_1 \end{array}$$

(13.1*c*)
$$\begin{array}{c} \textit{they are not arriving} \\ \pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{j}^\ell\mathbf{j}'\mathbf{p}_1^\ell)(\mathbf{p}_1\mathbf{p}_1^\ell)(\mathbf{p}_1\ \mathbf{i}'^\ell\mathbf{i}) \quad \rightarrow \mathbf{s}_1 \end{array}$$

(13.1*d*)
$$\begin{array}{c} \textit{they are not green} \\ \pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{j}^\ell\mathbf{i}\mathbf{a}^\ell)(\mathbf{a}\mathbf{a}^\ell)\mathbf{a} \quad \rightarrow \mathbf{s}_1 \end{array}$$

(13.1*e*)
$$\begin{array}{c} \textit{they do not arrive} \\ \pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{i}^\ell)\ (\mathbf{i}\ \mathbf{i}^\ell)\ \mathbf{i} \quad \rightarrow \mathbf{s}_1 \end{array}$$

but nowadays we do not say

(13.2)
$$\begin{array}{c} ^*\textit{they arrive not} \\ \pi_2\ (\pi_2^r\mathbf{s}_1\mathbf{j}^\ell\mathbf{i})(xx^\ell) \quad \not\rightarrow \mathbf{s}_1 \end{array}$$

although the corresponding sentence would be acceptable in German.

Negated modals and auxiliaries may be contracted:

will not $\rightarrow$ *won't*,
have not $\rightarrow$ *haven't*,
are not $\rightarrow$ *aren't*,
do not $\rightarrow$ *don't*.

These contracted forms have the same type as the verbs before being negated, e.g.

$$\begin{array}{c} \textit{will not} \ \rightarrow \ \textit{won't} \\ (\pi^r \mathbf{s}_1 \mathbf{j}^\ell)(\mathbf{j}\mathbf{j}^\ell) \ \rightarrow \ \pi^r \mathbf{s}_1 \mathbf{j}^\ell \end{array} \tag{13.3}$$

and Metarule 12.2 applies to them, e.g.

$$\begin{array}{c} \textit{won't they arrive ?} \\ \underbrace{(\mathbf{q}_1 \mathbf{j}^\ell \pi^\ell)}\ \underbrace{\pi_2}\ \underbrace{\mathbf{i}} \qquad \rightarrow \mathbf{q}_1 \end{array} \tag{13.4}$$

14. Wh-questions.

Recall that Metarule 12.2 was restricted to modal or auxiliary verbs, hence it does not admit (12.3) or

$$\begin{array}{c} {}^*\textit{arrives he} \\ \underbrace{(\mathbf{q}_1 \hat{\pi}_3^\ell \mathbf{j}^\ell \mathbf{i})}\ \underbrace{\pi_3} \end{array} \tag{14.1}$$

which will however be admitted as a pseudo-sentence later to analyze "*who arrives*?" It is convenient to assign the new type $\mathbf{q}_1 \hat{\pi}_3^\ell \mathbf{j}^\ell \mathbf{i}$ to *arrives* as long as $\pi_3 \not\rightarrow \hat{\pi}_3$. Here

$$\hat{\pi}_3 = \ \text{type of } \textit{pseudo} - \textit{subject},$$

and we postulate

$$\hat{\pi}_3 \rightarrow \pi_3 \not\rightarrow \hat{\pi}_3,$$

forbidding the contraction $\hat{\pi}_3^\ell \pi_3 \rightarrow 1$, which would wrongly imply

$$\pi_3 \rightarrow \hat{\pi}_3 \hat{\pi}_3^\ell \pi_3 \ \rightarrow \ \hat{\pi}_3.$$

The new type of *arrives* allows us to introduce the wh-question

$$\begin{array}{c} \textit{who arrives today ?} \\ \underbrace{(\overline{\mathbf{q}} \hat{\pi}_3^{\ell\ell} \mathbf{q}^\ell)}\ \underbrace{(\mathbf{q}_1 \hat{\pi}_3^\ell \mathbf{j}^\ell}\ \underbrace{[\mathbf{i})}\ \underbrace{(\mathbf{i}^r \mathbf{i})} \ \rightarrow \overline{\mathbf{q}} \end{array} \tag{14.2}$$

where

$\overline{\mathbf{q}}$ = question (including wh-question),

$\mathbf{q}$ = yes-or-no question when tense is irrelevant

and

$$\mathbf{q}_j \ \rightarrow \ \mathbf{q} \ \rightarrow \ \overline{\mathbf{q}}.$$

We may also put

$$\hat{\pi}_2 = \ \text{plural pseudo-subject}$$

and analyze

$$\begin{array}{c} \textit{whose parents arrive today ?} \\ \underbrace{(\overline{\mathbf{q}} \hat{\pi}_2^{\ell\ell} \mathbf{q}^\ell x^\ell)}\ \underbrace{\mathbf{p}}\ \underbrace{(\mathbf{q}_1 \hat{\pi}_2^\ell \mathbf{j}^\ell}\ \underbrace{[\mathbf{i})}\ \underbrace{(\mathbf{i}^r \mathbf{i})} \end{array} \tag{14.3}$$

where $x = \mathbf{p}, \mathbf{c}$ or $\mathbf{m}$.

However, to analyze

$$\begin{array}{c} \textit{who has arrived ?} \\ (\overline{\mathbf{q}}\hat{\pi}_3^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_1\hat{\pi}_3^\ell\mathbf{p}_2^\ell)(\mathbf{p}_2\mathbf{j}^\ell\mathbf{i}) \ \ \to \overline{\mathbf{q}} \end{array} \tag{14.4}$$

we require yet another type $\mathbf{q}_1\hat{\pi}_3^\ell\mathbf{p}_2^\ell$ for *has*. The new types for *arrives* in (14.2), *arrive* in (14.3) and *has* in (14.4) are all justified by the following:

Metarule 14.5. If the finite form of a verb has type $\pi_k^r\mathbf{s}_j x^\ell$ $(x = 1, \mathbf{j}, \mathbf{p}_2, \mathbf{p}_1, \mathbf{i}, \mathbf{o}, \cdots)$ in a statement, then it may have type $\mathbf{q}_j\hat{\pi}_k^\ell x^\ell$ in a wh-question.

We will test the case $x = \mathbf{o}$ with the question

$$\begin{array}{c} \textit{who ate the apple ?} \\ (\overline{\mathbf{q}}\hat{\pi}_3^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\hat{\pi}_3^\ell\mathbf{o}^\ell)(\overline{\mathbf{c}}\mathbf{c}^\ell)\ \mathbf{c} \ \ \to \overline{\mathbf{q}} \end{array} \tag{14.6}$$

where

$\mathbf{o}$ = (type of) direct object

$\overline{\mathbf{c}}$ = complete singular noun phrase

and we postulate

$$\overline{\mathbf{c}} \to \mathbf{o},\ \hat{\pi}_3.$$

Asking for the direct object instead, we are led to the example [C1957 (68)]:

$$\begin{array}{c} \textit{what did John eat} - \textit{?} \\ (\overline{\mathbf{q}}\mathbf{o}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\mathbf{i}^\ell\pi_3^\ell)\ \mathbf{n}\ (\mathbf{i}\mathbf{o}^\ell) \ \to \ \overline{\mathbf{q}} \end{array} \tag{14.7}$$

where

$$\mathbf{n} = \text{ name}$$

and we postulate

$$\mathbf{n} \to \pi_3, \mathbf{o}.$$

When I first proposed pregroup grammars in 1998, Michael Moortgat asked: What if we add an adverb such as *today* to (14.1)? I now propose

Metarule 14.8. Every transitive verb of type $\mathbf{i}\mathbf{o}^\ell$ may also be assigned the type $\mathbf{i}\hat{\mathbf{o}}^\ell\mathbf{i}^\ell\mathbf{i}$, where

$$\hat{\mathbf{o}} \ \to \ \mathbf{o} \ \not\to \ \hat{\mathbf{o}}.$$

We should also refine the type of the object question word as follows:

$$\textit{what},\ \textit{whom}:\ \overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\mathbf{q}^\ell.$$

This is a refinement, since $\hat{\mathbf{o}}^{\ell\ell} \to \mathbf{o}^{\ell\ell}$ by (2.2). We then obtain

$$\begin{array}{c} \textit{what did John eat today ?} \\ (\overline{\mathbf{q}}\hat{\mathbf{o}}^{\ell\ell}\mathbf{q}^\ell)(\mathbf{q}_2\mathbf{i}^\ell\pi^\ell)\ \mathbf{n}\ (\mathbf{i}\hat{\mathbf{o}}^\ell\mathbf{i}^\ell[\mathbf{i}])(\mathbf{i}^r\mathbf{i}) \end{array} \tag{14.9}$$

with a left square bracket to prevent premature contraction.

15. Discontinuous dependencies.

In [C1957 (82)], Chomsky compares

(15.1a) *the police brought the criminal in*

(b) .....*him* .....

(c) *the police brought in the criminal*

(d) * ..... *him*

This raises the question why (d) is not allowed. We resolve this problem by assigning two different types to *bring in*:

$$\begin{array}{ccccc} \textit{bring} & - & \textit{in}\ , & \textit{bring in} \\ (\mathbf{i}\delta^\ell\mathbf{o}^\ell) & & \delta & (\mathbf{i}\hat{\mathbf{o}}^\ell\delta^\ell)\delta \to \mathbf{i}\hat{\mathbf{o}}^\ell \end{array}$$

yielding

(15.2*a*)
$$\begin{array}{c} \textit{bring in the criminal} \\ \underbrace{(\mathbf{i}\hat{\mathbf{o}}^\ell\delta^\ell)\ \delta\ (\overline{\mathbf{c}}\mathbf{c}^\ell)}\ \underbrace{\mathbf{c}} \ \to\ \mathbf{i} \end{array}$$

(15.2*b*)
$$\begin{array}{c} {}^*\textit{bring him in} \\ \underbrace{(\mathbf{i}\hat{\mathbf{o}}^\ell\delta^\ell)\ \delta}\ \underbrace{\mathbf{o}} \ \not\to\ \mathbf{i} \end{array}$$

provided we postulate

$$\overline{\mathbf{c}} \to \hat{\mathbf{o}} \to \mathbf{o} \not\to \hat{\mathbf{o}}\ .$$

To justify the double type assignment to *bring* with a detachable suffix of type δ, we rely on the special case $x = \delta$ of the following general metarule:

Metarule 15.3. If the infinitive of a verb has type $\mathbf{i}x^\ell\mathbf{o}^\ell$, where $x = \delta, \mathbf{j}, a, \mathbf{i}^r\mathbf{i}, \mathbf{j}^r\mathbf{j}, \cdots$, then it can also have type $\mathbf{i}\hat{\mathbf{o}}^\ell x^\ell$.

The case $x = \mathbf{j}$ is illustrated by the following examples:

(15.4*a*)
$$\begin{array}{c} \textit{let the girl go} \\ \underbrace{(\mathbf{i}\mathbf{j}^\ell\mathbf{o}^\ell)(\overline{\mathbf{c}}\mathbf{c}^\ell)\mathbf{c}\ \mathbf{i}} \ \to \mathbf{i} \end{array}$$

(15.4*b*)
$$\begin{array}{c} \textit{let her go} \\ \underbrace{(\mathbf{i}\mathbf{j}^\ell\mathbf{o}^\ell)\ \mathbf{o}\ \mathbf{i}} \ \to \mathbf{i} \end{array}$$

(15.4*c*)
$$\begin{array}{c} \textit{let go the girl} \\ \underbrace{(\mathbf{i}\hat{\mathbf{o}}^\ell\mathbf{j}^\ell)\ \mathbf{i}(\overline{\mathbf{c}}\mathbf{c}^\ell)}\ \underbrace{\mathbf{c}} \ \to \mathbf{i} \end{array}$$

(15.4*d*)
$$\begin{array}{c} {}^{*}\textit{let go her} \\ (\mathbf{i}\hat{\mathbf{o}}^{\ell}\mathbf{j}^{\ell}\ \mathbf{i}\quad \mathbf{o} \quad \not\to \mathbf{i} \end{array}$$

For the case $x = \mathbf{a}$, consider

(15.5a) make the promise good
(b) make it good
(c) make good the promise
(d) *make good it

The case $x = \mathbf{i}^r\mathbf{i}$ allows us to re-analyze (10.2f) as

(15.6)
$$\begin{array}{c} \textit{they were seen today} \\ \pi_2(\pi_2^r\mathbf{s}_2\mathbf{j}^{\ell}\mathbf{i}'\hat{\mathbf{o}}^{\ell\ell}\mathbf{p}_2^{\ell})(\mathbf{p}_2\mathbf{j}'^{\ell}\mathbf{i}\hat{\mathbf{o}}^{\ell}\mathbf{i}^{\ell}[\mathbf{i}])(\mathbf{i}^r\mathbf{i}) \quad \to \mathbf{s}_2 \end{array}$$

Since $(\mathbf{i}^r\mathbf{i})^{\ell} = \mathbf{i}^{\ell}\mathbf{i}^{r\ell} = \mathbf{i}^{\ell}\mathbf{i}$.

16. Concluding remarks.

The primary aim of this article was to provide an easy introduction to pregroup grammars at the hand of a few examples, which had been used by Chomsky in 1957 to challenge the linguistic orthodoxy of the time.

I took the opportunity to make some small improvements on earlier treatments of pregroup grammar [L1999, 2004], mainly by incorporating some intermediate infinitival types between the type **i** of *sleep* and the type **j** of *have slept*. As a result, it is no longer necessary to stipulate that certain auxiliary verbs lack some conjugational forms.

In going over Chomsky's old material, my present objective was to see how questions of syntax and morphology can be handled formally in the framework of pregroup grammar by appropriate type assignments to words in the mental dictionary. However, I could not help making some informal observations concerning the syntax-semantics interface as well, admittedly ignoring the intense debate of this topic in linguistic circles during the last half century.

I must confess that the pregroup approach is not quite as simple as Chomsky's original generative-transformational one, only that it aims to complement the latter by providing a model for subconscious computations (see [L2004]). A comparison with Chomsky's more recent theories is left to the future.

17. Mathematical background.

Mathematically trained readers may be interested in the following material, but others are invited to skip it.

Definition 17.1. A *pregroup* is a partially ordered monoid in which each element has both a *left adjoint* x^{ℓ} and a *right adjoint* x^r such that

$$x^{\ell}x \to 1 \to xx^{\ell}, \quad xx^r \to 1 \to x^r x.$$

The terminology is borrowed from category theory. It can be shown that adjoints are unique and that

(17.2)
$$\begin{array}{ll} 1^{\ell} = 1 = 1^r, & x^{\ell r} = x = x^{r\ell}, \\ (xy)^{\ell} = y^{\ell}x^{\ell}, & (xy)^r = y^r x^r; \end{array}$$

(17.3) if $x \to y$ then $y^\ell \to x^\ell$ and $y^r \to x^r$.

Our types, i.e. strings of simple types, are easily seen to form a pregroup, with concatenation serving as the monoid operation. In fact, it is *the* pregroup *freely generated* by the partially ordered set of basic types. The following so-called Switching Lemma was established in [L1999].

Lemma 17.4. When showing that $a \to b$ in the pregroup freely generated by the partially ordered set of basic types, one may assume, without loss of generality, that all contractions $x^\ell x \to 1$ and $xx^r \to 1$ precede all expansions $1 \to xx^\ell$ and $1 \to x^r x$.

For linguistic purposes, it usually suffices to calculate $a \to b$ when b is a simple or even basic type, e.g. that of a sentence. As a consequence of Lemma 17.4, expansions need not appear at all in such a calculation. Still, expansions are useful in proving that (17.2) and (17.3) hold in any pregroup. Not surprisingly, Harris [H1966, 1968] had no need for expansions at all.

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FOOTNOTES

[1)] This material was first presented at a McGill seminar in early 2005 and at a conference in Chieti in May 2005. The author acknowledges support from the Social Sciences and Humanities Research Council of Canada.

[2)] It might be of interest to know whether sufferers from Asperger's syndrome, alleged to be unaware of other people's mental or emotional states, can come up with the same list (9.1).

[3)] Here the copula *be* of type $\mathbf{ja}^\ell$ is also considered to be an auxiliary verb.