Proofs and Dialogue : the Ludics view

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Where Ludics come from?

Ludics is a theory elaborated by J-Y. Girard in order to rebuild logic starting from the notion of *interaction*.
It starts from the concept of **proof**, as was investigated in the framework of **Linear Logic**:

- **Linear Logic** *may be polarized* ($\rightarrow$ negative and positive rules)
- **Linear Logic** leads to the important notion of **proof-net** ($\rightarrow$ *being a proof is more a question of connections than a question of formulae to be proven*) $\rightarrow$ **loci**

Polarization

Results on polarization come from those on **focalization** (Andréoli, 1992)

- some connectives are *deterministic* and *reversible* (= **negative** ones) : their right-rule, which may be read in both directions, may be applied in a deterministic way:

Example

$$\frac{\vdash A, B, \Gamma}{\vdash A \wp B, \Gamma}[\wp] \qquad \frac{\vdash A, \Gamma \quad \vdash B, \Gamma}{\vdash A \& B, \Gamma}[\&]$$

Polarization

- the other connectives are *non-deterministic* and *non-reversible* (= **positive** ones) : their right-rule, which cannot be read in both directions, may not be applied in a deterministic way (from bottom to top, there is a choice to be made) :

Example

$$\frac{\vdash A, \Gamma \quad \vdash B, \Gamma'}{\vdash A \otimes B, \Gamma, \Gamma'}[\otimes] \qquad \frac{\vdash A, \Gamma}{\vdash A \oplus B, \Gamma}[\oplus_g] \qquad \frac{\vdash B, \Gamma}{\vdash A \oplus B, \Gamma}[\oplus_d]$$

The Focalization theorem

- every proof may be put in such a form that :
  - while there are negative formulae in the (one-sided) sequent to prove, choose one of them at random,
  - as soon as there are no longer negative formulae, choose a positive one and then continue to **focalize** it
- we may consider positive and negative "blocks" → **synthetic connectives**
- convention : the *negative* formulae will be written as positive but on the left hand-side of a sequent → **fork**

Hypersequentialized Logic

Formulae:

$$F = O|1|P|(F^{\perp} \otimes \cdots \otimes F^{\perp}) \oplus \cdots \oplus (F^{\perp} \otimes \cdots \otimes F^{\perp})|$$

Rules :

- axioms :

$$\frac{}{P \vdash P, \Delta} \qquad \frac{}{\vdash 1, \Delta} \qquad \frac{}{O \vdash \Delta}$$

- logical rules :

$$\frac{\vdash A_{11}, \ldots, A_{1n_1}, \Gamma \quad \ldots \quad \vdash A_{p1}, \ldots, A_{pn_p}, \Gamma}{(A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}) \vdash \Gamma}$$

$$\frac{A_{i1} \vdash \Gamma_1 \quad \ldots A_{in_i} \vdash \Gamma_p}{\vdash (A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}), \Gamma}$$

  where $\cup\Gamma_k \subset \Gamma$[1] and, for $k, l \in \{1, \ldots p\}$, $\Gamma_k \cap \Gamma_l = \emptyset$.

- cut rule :

$$\frac{A \vdash B, \Delta \qquad B \vdash \Gamma}{A \vdash \Delta, \Gamma}$$

Remarks

- all propositional variables P are supposed to be **positive**
- formulae connected by the positive $\otimes$ and $\oplus$ are **negative** (positive formulae are maximal positive decompositions)
- $(\ldots \otimes \ldots \otimes \ldots) \oplus (\ldots \otimes \ldots \otimes \ldots)\ldots \oplus (\ldots \otimes \ldots \otimes \ldots)$ is not a restriction because of distributivity $((A \oplus B) \otimes C \equiv (A \otimes C) \oplus (B \otimes C))$

Interpretation of the rules

- **Positive** rule :
  1. choose $i \in \{1, ..., p\}$ (a $\oplus$-member)
  2. then decompose the context Γ into disjoint pieces
- **Negative** rule :
  1. nothing to choose
  2. simply enumerates all the possibilities

First interpretation, as **questions** :

- **Positive** rule : to choose a component where to answer
- **Negative** rule : the range of possible answers

The daimon

Suppose we use a rule:

$$\frac{}{\vdash \Gamma}\ (\mathbf{stop}!)$$

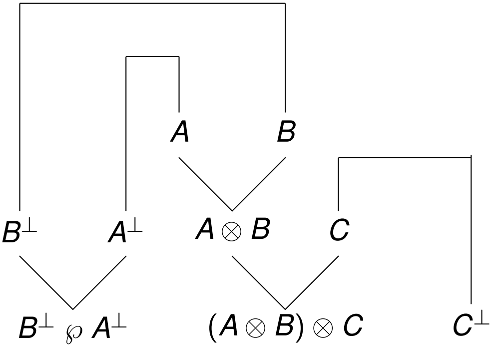
for any sequence Γ, *that we use when cannot do anything else...*

- the system now “accepts” proofs which are not real ones
- if **(stop!)** is used, this is precisely because... the process does not lead to a proof!
- **(stop!)** is a **paralogism**
- the proof ended by **(stop!)** is a **paraproof**
- cf. (in classical logic) it could give a distribution of truth-values which gives a counter-example (therefore also: *counter-proof*)

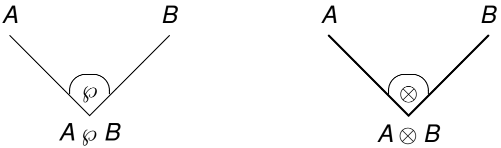
A reminder of proof-nets

$$\vdash A^{\perp} \wp B^{\perp}, (A \otimes B) \otimes C, C^{\perp}$$

$$\begin{array}{c}
\dfrac{\dfrac{\vdash A, A^{\perp} \qquad \vdash B, B^{\perp}}{\vdash A \otimes B, A^{\perp}, B^{\perp}} \qquad \vdash C, C^{\perp}}{\vdash (A \otimes B) \otimes C, A^{\perp}, B^{\perp}, C^{\perp}} \\ \hline\hline
\vdash A^{\perp}, B^{\perp}, (A \otimes B) \otimes C, C^{\perp} \\ \hline
\vdash A^{\perp} \wp B^{\perp}, (A \otimes B) \otimes C, C^{\perp}
\end{array}
\qquad
\begin{array}{c}
\dfrac{\dfrac{\dfrac{\vdash A, A^{\perp} \qquad \vdash B, B^{\perp}}{\vdash A \otimes B, A^{\perp}, B^{\perp}}}{\vdash A \otimes B, A^{\perp} \wp B^{\perp}} \qquad \vdash C, C^{\perp}}{\vdash (A \otimes B) \otimes C, A^{\perp} \wp B^{\perp}, C^{\perp}} \\ \hline\hline
\vdash A^{\perp} \wp B^{\perp}, (A \otimes B) \otimes C, C^{\perp}
\end{array}$$

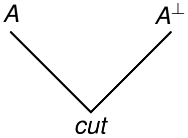

A
B
B⊥
A⊥
A ⊗ B
C
B⊥ ℘ A⊥
(A ⊗ B) ⊗ C
C⊥

1. ”par” and ”tensor” links:



2. ”Axiom” link

3. “Cut” link

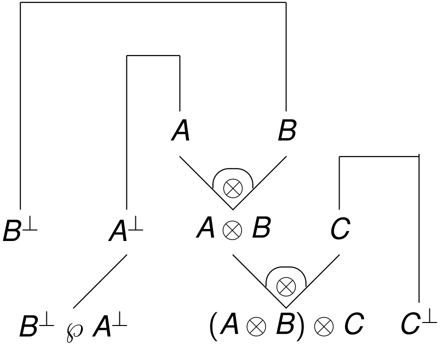


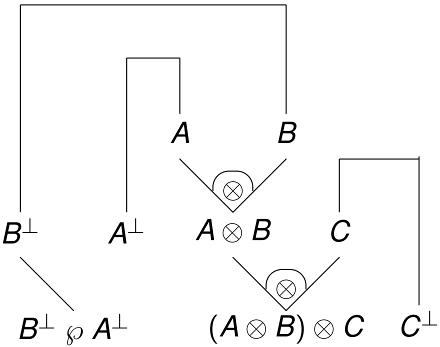
We define a *proof structure* as any such a graph built only by means of these links such that each formula is the conclusion of exactly one link and the premiss of at most one link.

Criterion

Definition (Correction criterion)

correction criterion A proof structure is a proof net if and only if the graph which results from the removal, for each $\wp$ link ("par" link) in the structure, of one of the two edges is connected and has no cycle (that is in fact a tree).


A
B
B⊥
A⊥
A ⊗ B
C
B⊥ ℘ A⊥
(A ⊗ B) ⊗ C
C⊥


A
B
⊗
B⊥
A⊥
A ⊗ B
C
⊗
B⊥ ℘ A⊥
(A ⊗ B) ⊗ C
C⊥

Loci

Rules do not apply to **contents** but to **addresses**

Example

$$\dfrac{\vdash e^{\perp}, l \qquad \dfrac{\vdash e^{\perp}, c}{\vdash e^{\perp}, c \oplus d}}{\dfrac{\vdash e^{\perp}, l \& (c \oplus d)}{\vdash e^{\perp} \wp (l \& (c \oplus d))}} \qquad \dfrac{\vdash e^{\perp}, l \qquad \dfrac{\vdash e^{\perp}, d}{\vdash e^{\perp}, c \oplus d}}{\dfrac{\vdash e^{\perp}, l \& (c \oplus d)}{\vdash e^{\perp} \wp (l \& (c \oplus d))}}$$

under a focused format :

$$\dfrac{\vdash e^{\perp}, l \qquad \dfrac{c^{\perp} \vdash e^{\perp}}{\vdash e^{\perp}, c \oplus d}}{e \otimes (l^{\perp} \oplus (c \oplus d)^{\perp}) \vdash} \qquad \dfrac{\vdash e^{\perp}, l \qquad \dfrac{d^{\perp} \vdash e^{\perp}}{\vdash e^{\perp}, c \oplus d}}{e \otimes (l^{\perp} \oplus (c \oplus d)^{\perp}) \vdash}$$

with only loci:

$$\dfrac{\vdash \xi 1, \xi 2 \qquad \dfrac{\xi.3.1 \vdash \xi 1}{\vdash \xi.1, \xi.3}}{\xi \vdash} \qquad\qquad \dfrac{\vdash \xi 1, \xi 2 \qquad \dfrac{\xi.3.2 \vdash \xi 1}{\vdash \xi.1, \xi.3}}{\xi \vdash}$$

Rules

Definition

positive rule

$$\frac{\ldots \quad \xi \star i \vdash \Lambda_i \quad \ldots}{\vdash \xi, \Lambda} \, (+, \xi, I)$$

- $i \in I$
- all Λ_i's pairwise disjoint and included in Λ

Definition

negative rule

$$\frac{\ldots \quad \vdash \xi \star J, \Lambda_J \quad \ldots}{\xi \vdash \Lambda} \, (-, \xi, \mathcal{N})$$

daimon

Dai

$$\frac{}{\vdash \Lambda} \dagger$$

- it is a positive rule (something we choose to do)
- it is a *paraproof*

Is there a identity rule?

- No, properly speaking (since there are lo longer **atoms**!)
- two *loci* cannot be identified
- there only remains the opportunity to recognize that two sets of addresses correspond to each other by displacement : $\mathcal{F}ax$

$$Fax_{\xi,\xi'} = \cfrac{\cfrac{\cfrac{\ldots \quad Fax_{\xi_{i1},\xi'_{i1}} \quad \ldots}{\ldots\xi' \star i \vdash \xi \star i \ldots}}{\ldots \quad \vdash \xi \star J_1, \xi' \quad \ldots} (+, \xi', J_1)}{\xi \vdash \xi'} (-, \xi, \mathcal{P}_f(\mathbb{N}))$$

Infinite proofs

- $\mathcal{F}ax$.... is **infinite**! (cf. the directory $\mathcal{P}_f(\mathbb{N})$)
- it provides a way to explore any "formula" (a tree of addresses) at any depth

Designs

Definition

A **design** is a tree of **forks** $\Gamma \vdash \Delta$ the root of which is called the **base** (or conclusion), which is built only using :

- daimon
- positive rule
- negative rule

a design...

Example

$$\dfrac{\dfrac{011 \vdash \quad 012 \vdash 02}{\vdash 01, 02}(+, 01, \{1,2\}) \quad \dfrac{031 \vdash \quad 033 \vdash 01}{\vdash 01, 03}(+, 03, \{1,3\})}{\dfrac{0 \vdash}{\vdash <>}(+, <>, \{0\})}(-, 0, \{\{1,2\}, \{1,3\}\})$$

- a negative step gives a fixed **focus** and a set of **ramifications**,
- on such a basis, a positive step chooses a **focus** and a **ramification**

An illustration

- positive rule : a **question** (*where will you go next week ?*)
- negative rule : a scan of possible answers is provided, (*Roma* and *Naples* or *Rome* and *Florence*)
- in case of the choice 1 : positive rule on the base ”Roma”, new questions (*with whom?* and *by what means?*)
- in case of choice 2 : positive rule on the base ”Florence”, new questions (*with whom?* and *how long will you stay?*)

Normalization

- no explicit cut-rule in Ludics
- but an implicit one : the meeting of same addresses with opposite polarity

Example

$$\dfrac{\dfrac{\dfrac{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \qquad \dfrac{\dfrac{\vdots}{\vdash \xi 21} \quad \dfrac{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{\vdash {\color{red}\xi}} \qquad \dfrac{\dfrac{\dfrac{\vdots}{\xi 11 \vdash \xi 2} \quad \dfrac{\vdots}{\xi 12 \vdash}}{\vdash \xi 1, \xi 2}}{{\color{red}\xi} \vdash}$$
$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$
\dfrac{\dfrac{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash}
\qquad
\dfrac{\dfrac{\vdots}{\xi 11 \vdash \xi 2} \quad \dfrac{\vdots}{\xi 12 \vdash}}{\vdash \xi 1, \xi 2}
\qquad
\dfrac{\dfrac{\vdots}{\vdash \xi 21} \quad \dfrac{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}
$$

which is rewritten in:

$$\underline{\begin{array}{c}\vdots\\ \xi 12 \vdash\end{array} \quad \begin{array}{c}\vdots\\ \vdash \xi 12,\end{array}} \ \xi 11 \ \underline{\qquad\qquad \begin{array}{c}\vdots\\ \xi 11 \vdash\end{array}} \ \xi 2 \ \underline{\qquad \dfrac{\begin{array}{c}\vdots\\ \vdash \xi 21\end{array} \quad \begin{array}{c}\vdots\\ \vdash \xi 22, \xi 23\end{array}}{\xi 2 \vdash}}$$

And so on . . .

When the interaction meets the daimon, it converges. The two interacting designs are said **orthogonal**

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \quad \dfrac{\dfrac{\vdots}{\vdash \xi 21} \quad \dfrac{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{\vdash \xi} \qquad \dfrac{\dfrac{}{\vdash \xi 1, \xi 2}\dagger}{\xi \vdash}}{}$$

Otherwise the interaction is said **divergent**.

$$\dfrac{\dfrac{\dfrac{\overset{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \qquad \dfrac{\overset{\vdots}{\vdash \xi 21} \qquad \overset{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{\vdash \xi} \qquad \dfrac{\overset{\vdots}{\vdash \xi 1, \xi 2, \xi 3}}{\xi \vdash}}{}$$

Normalization, formally - 1- Closed nets

Namely, a **closed net** consists in a cut between the two following designs:

$$\cfrac{\cfrac{\begin{array}{c}\mathcal{D}\\ \vdots\end{array}}{\vdash \xi}\,\kappa \qquad \cfrac{\begin{array}{c}\mathcal{E}\\ \vdots\end{array}}{\xi \vdash}\,(\xi, \mathcal{N})}{}$$

Orthogonality

- if κ is the daimon, then the normalized form is :

$$\frac{}{\vdash} \dagger$$

(this normalised net is called dai)

- if $\kappa = (\xi, I)$, then if $I \notin \mathcal{N}$, normalization fails,
- if $\kappa = (\xi, I)$ and $I \in \mathcal{N}$, then we consider, for all $i \in I$ the design $\mathcal{D}_i$, sub-design of $\mathcal{D}$ of basis $\xi \star i \vdash$, and the sub-design $\mathcal{E}'$ of $\mathcal{E}$, of basis $\vdash \xi \star I$, and we replace $\mathcal{D}$ and $\mathcal{E}$ by, respectively, the sequences of $\mathcal{D}_i$ and $\mathcal{E}'$.

In other words, the initial net is replaced by :

$$
\dfrac{\dfrac{\mathcal{D}_{i_1}\ \vdots}{\xi \star i_1 \vdash} \quad \ldots \quad \dfrac{\mathcal{E}'\ \vdots}{\vdash \xi \star i_1, \ldots, \xi \star i_n} \quad \dfrac{\mathcal{D}_{i_n}\ \vdots}{\xi \star i_n \vdash}}{}
$$

with a cut between each $\xi \star i_j \vdash$ and the corresponding "formula" $\xi \star i_j$ in the design $\mathcal{E}'$

The separation theorem

Theorem

If $\mathcal{D} \neq \mathcal{D}'$ then there exists a counterdesign $\mathcal{E}$ which is orthogonal to one of $\mathcal{D}, \mathcal{D}'$ but not to the other.

Hence the fact that: the objects of ludics are completely defined by their interactions

- a design $\mathcal{D}$ inhabits its **behaviour** (= like its **type**)
- a **behaviour** is a set of designs which is stable by **bi-orthogonality** ($\mathbb{G} = \mathbb{G}^{\perp\perp}$)

The game aspect

A slight change of vocabulary:

step in a proof	action	
positive step	**positive** action	$(+, \xi, I)$
negative step	**negative** action	$(-, \zeta, J)$
branch of a design	*play in a game*	**chronicle**
design	*strategy*	design (**dessein**) as a set of chronicles

Example

$$\dfrac{\dfrac{011 \vdash \quad 012 \vdash 02}{\vdash 01, 02}(+, 01, \{1,2\}) \qquad \dfrac{}{\vdash 01, 03}\dagger}{\dfrac{0 \vdash}{\vdash <>}(+, <>, \{0\})}(-, 0, \{\{1,2\},\{1,3\}\})$$

Example

$(+, <>, 0), (-, 0, \{1, 2\}), (+, 01, \{1, 2\})$
$(+, <>, 0), (-, 0, \{1, 3\}), (+, \dagger)$

Remarks

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics:
  - proof vs counter - proof
  - processes anchored on A vs processes anchored on $\neg A$
- analogies:
  - argumentation vs refutation

In Ludics a "proof" is completely defined by its interactions.

Dialogue in Ludics

The archetypal figure of interaction is provided by two intertwined processes the successive times of which, alternatively positive and negative, are opposed by pairs.

Ludics	**Dialogue**
Positive rule	**performing** an intervention or **commiting oneself** (Brandom)
Negative rule	**recording** or awaiting or **being authorized**
Daïmon	giving up or ending an exchange

The positive rule : “Proof” reading

$$\dfrac{\begin{matrix}\vdots & \vdots & \vdots \\ 01 \vdash \Delta_1 & 02 \vdash \Delta_2 & 03 \vdash \Delta_3\end{matrix}}{\vdash 0, \Delta}$$

$$\dfrac{\begin{matrix}\vdots & \vdots & \vdots \\ P_1 \vdash \Delta_1 & P_2 \vdash \Delta_2 & P_3 \vdash \Delta_3\end{matrix}}{\vdash P, \Delta}$$

- You **decide to defend** a formula P in the context Δ, (*you do not know exactly what P is* : it may be equal to $Q_1^\perp \otimes Q_2^\perp$ or equal to $R_1^\perp$ or to $P_1^\perp \otimes P_2^\perp \otimes P_3^\perp$ or $\ldots$;
- You choose one of these possibilities : $P_1^\perp \otimes P_2^\perp \otimes P_3^\perp$;
- You are **committed** to $P_1^\perp$ and $P_2^\perp$ and $P_3^\perp$.

The positive rule: “Dialogue reading”

$$\dfrac{\begin{matrix}\vdots & \vdots & \vdots \\ 01 \vdash \Delta_1 & 02 \vdash \Delta_2 & 03 \vdash \Delta_3\end{matrix}}{\vdash 0, \Delta}$$

- At this time of the process you dispose of a set of *loci* in ”positive” position.For example during a conversation, it is your turn of speech
- You have to choose a focus. You decide to speak about your next holidays. (here denoted by ’0’). This locus is made to vary across the various manners a given theme may be addressed. “This year, for my holidays, I will go to the Alps (01) with friends (02) and by walking (03).

The negative rule: "Proof" reading

$$\dfrac{\begin{array}{c}\vdots\\ \vdash 0\star I_1,\Gamma\end{array}\quad\ldots\quad\begin{array}{c}\vdots\\ \vdash 0\star I_k,\Gamma\end{array}\quad\ldots\quad\begin{array}{c}\vdots\\ \vdash 0\star I_n,\Gamma\end{array}}{0\vdash\Gamma}$$

Proof reading $$\dfrac{\begin{array}{c}\vdots\\ \vdash Q_1,Q_2,\Gamma\end{array}\quad\ldots\quad\begin{array}{c}\vdots\\ \vdash R_1,\Gamma\end{array}\quad\ldots\quad\begin{array}{c}\vdots\\ \vdash P_1,P_2,P_3,\Gamma\end{array}}{\mathbf{P}\vdash\Gamma}$$

- You want **to refute** the formula P.
  or defend the formula $P^{\perp} = (Q_1 \wp Q_2)\&(R_1)\&(P_1 \wp P_2 \wp P_3)$.
- You have to be ready to sustain this contradiction for all possible decompositions of P.

The negative rule: “Dialogue” reading

$$\frac{\begin{array}{c}\vdots\\ \vdash 0 \star I_1, \Gamma\end{array} \quad \ldots \quad \begin{array}{c}\vdots\\ \vdash 0 \star I_k, \Gamma\end{array} \quad \ldots \quad \begin{array}{c}\vdots\\ \vdash 0 \star I_n, \Gamma\end{array}}{0 \vdash \Gamma}$$

This represents a receptive attitude : the locus is the one which has been selected in the other process (by your addressee). The different branches in your process represent a survey of all the various ways you may consider as possible ways to address this theme.

Convergence and divergence

- *Convergence in dialogue holds as long as expectations from one speaker contain commitments of the other* (*pragmatics*: *"Be relevant!"* replaced by *"Keep convergent!"*)
- **orthogonality** = **private** communication
- **non-orthogonality** : normalization may yield *side effects* : public results of communication

$$\dfrac{\begin{matrix}\vdots & \vdots & \vdots \\ P_1 \vdash \Delta_1 & P_2 \vdash \Delta_2 & P_3 \vdash \Delta_3\end{matrix}}{\vdash \mathbf{P}, \Delta}$$

I **commit myself**
to speak of P_1, P_2, P_3

$$\dfrac{\begin{matrix}\vdots & & \vdots & & \vdots \\ \vdash Q_1, Q_2, \Gamma & \ldots & \vdash R_1, \Gamma & \ldots & \vdash P_1, P_2, P_3, \Gamma\end{matrix}}{\mathbf{P} \vdash \Gamma}$$

among **authorizations**
provided by interlocutor

The daimon rule

$$\frac{}{\vdash \Delta}\dagger$$

- In proof reading this represents the fact to abandon your proof search or your counter-model attempt.
- This represents the fact to close a dialogue (by means of some explicite signs : “well”, “OK”, . . . or implicitely because it is clear that an answer was given, an argument was accepted and so on. . .).

Examples

Example of two elementary dialogues:

Example

The first one is well formed:

- Have you a car?
- Yes,
- Of what mark?

$$\dfrac{\dfrac{\dfrac{}{\vdash}\dagger \qquad \dfrac{\dfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \sigma}}{\vdash \xi01, \sigma}}{\xi0 \vdash \sigma}\{\emptyset,\{1\}\}}{\vdash \xi, \sigma} \qquad \text{vs} \qquad \dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots \\ \vdash \xi010\end{array}}{\xi01 \vdash}}{\vdash \xi0}}{\xi \vdash}$$

Examples

The locus σ is a place for recording the answer:

Example

- Have you a car?
- Yes,
- Of what mark?
- Honda.

$$\dfrac{\dfrac{}{\vdash}\dagger \quad \dfrac{\dfrac{Fax_{\xi 010,\sigma}}{\xi 010 \vdash \sigma}}{\vdash \xi 01, \sigma}}{\dfrac{\xi 0 \vdash \sigma}{\vdash \xi, \sigma}}\{\emptyset,\{1\}\} \qquad \text{vs} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\xi 010k \vdash}{\vdash \xi 010}}{\xi 01 \vdash}}{\vdash \xi 0}}{\xi \vdash}$$

The interaction reduces to:

Example

$$\frac{\sigma k \vdash}{\vdash \sigma}$$

The mark of the car is “Honda”.
This “assertion” is recorded by the speaker.

It is the function of $\mathcal{F}ax$ to interact in such a way that the design anchored on ξ_{010} is transferred to the address σ, **thus providing the answer.**

The second dialogue is ill formed:

Example

- Have you a car?
- No, I have no car.
- * Of what mark?

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{Fax_{\xi 010,\sigma}}{\xi 010 \vdash \sigma}}{\vdash \xi 01, \sigma}}{\xi 0 \vdash \sigma}\{\{1\}\}}{\vdash \xi, \sigma}You_1 \quad \text{vs} \quad \dfrac{\dfrac{}{\vdash \xi 0}\emptyset}{\xi \vdash}$$

the dialogue fails because *YOU* did not planified a negative answer,

Modelling dialogue

Intervention of S	**Current state**	**Intervention of** A
$\mathfrak{S}_1$		
	$\mathfrak{E}_1 = \mathfrak{S}_1$	
		$\mathfrak{A}_2$
	$\mathfrak{E}_2 = [[\mathfrak{E}_1, \mathfrak{A}_2]]$	
$\mathfrak{S}_3$		
	$\mathfrak{E}_3 = [[\mathfrak{E}_2, \mathfrak{S}_3]]$	
$\vdots$	$\vdots$	$\vdots$

Further developments

- K. Terui's c-designs : **computational designs**
  - from *absolute* addresses to relative addresses : *variables* of designs
  - ramifications replaced by named actions with an arity
  - finite objects: generators, in case of infinite designs
  - c-designs are **terms** which generalize λ-terms(simultaneous and parallel reductions via several channels)
- inclusion of **exponentials** (authorizes replay)

The introduction of variables allows to deal with designs with variables which correspond to designs with partial information (the whole future may stay unknown)