Deducing Meaning in Linear Logic

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1 Introduction

Linear Logic has began to be used intensively in several directions for some years, in Linguistics. Among these directions, many works have been done in the spirit of Categorial Grammar (Moortgat 1988, Morrill 1994, Lecomte 1994), and others are related to the syntax-semantics interface in the LFG framework (Dalrymple et al. 1993, Dalrymple et al. 1995). What we intend to do in this paper is much more along the lines of the latter than of the former, even if we think possible to link the present conception of this interface also to categorial syntax. Nevertheless, the content of this paper is very different from the conception which is involved in the LFG approach. First of all, it deals more deeply with Lexical Semantics. Secondly, it aims at being rather neutral with regards to the adopted syntactic model. Our view could be applied for instance also to the Lexicon-Grammar approach (Gross, 1975). The only requirement being that complement places be correctly assigned by the syntactic analysis in any verbal clause. Thirdly, it adopts a deductive conception of thematic roles according to which roles have to be deduced instead of being postulated. Actually, verbal meanings are unfolded into primitive relations which support thematic roles as some of their places.
Like in the LFG approach, however, Linear Logic is used as a *glue language*. In many respects, it takes the place of the λ-calculus, which happens to be too limited for our purpose. For instance, the use of the λ-calculus requires strong assomptions on the order of complements, which are seldom met in reality (Reyle 1988, Dalrymple et al. 1993). A deductive framework is more convenient than the λ-calculus because we can use commutative connectives (like conjunction) in combining fragments of meaning instead of necessarily ordering them. Moreover, linear logic is more convenient than classical logic for many reasons:

- it provides more ways of combining meanings: let us recall that in Linear Logic, every connective (and, or) splits into two varieties: an additive one and a multiplicative one, and that, moreover, these connectives have rather simple interpretations in the coherent semantics (Girard, 1987, Girard, 1995),

- the linear implication (denoted by: $\multimap$) is interpreted (also in coherent spaces) as a special kind of function from a coherent space to another one: it is a function (called *linear* by Girard) which consumes exactly one information in order to produce another one. In logical terms, that translates into the fact that when premises are used once in a deduction, they are really consumed, or in other words, they can no longer be used in another deduction step. This allows us to speak in terms of *resources*. For instance, a verb "consumes" its subject exactly once, and each thematic role is also consumed exactly once.

- Linear Logic provides us with additive connectives, which can be interpreted as *choices*. The usual & is no longer seen as the traditional *and* but as an internal choice to make between two resources (it is called *with* and it is the additive conjunction). In the same vein, $\oplus$ (the additive disjunction) is seen as an external choice between resources. In other words, in A$\oplus$B, an external agent has made the choice between A and B and A$\oplus$B accepts anyone of the two choices.

Examples are taken from French but they are accompanied by their english translation.

2 Linking Syntax and Semantics

We assume here that a sentence is described by a syntactic representation, a semantic representation, and a mapping between them. Syntactic representations are built from syntactic constituents (NP, PP and the like). A lexical entry presents a thematic structure and one or more syntactic frame(s), with mappings between the former and each of the latter ones. These mappings make use of semantic features. For instance, let us compare:

1. *Marie a volé Pierre* (Marie stole from Pierre)
2. *Marie a volé le bijou* (Marie stole the jewel)
3. *Marie a volé le bijou à Pierre* (Marie stole the jewel from Pierre)

The sentences (1), (2) and (3) correspond to different syntactic frames and to different thematic structures. We can represent (1) and (2) by a common syntactic frame: N_0 V N_1, and (3) by the syntactic frame N_0 V N_1 à N_2. But in (1) and (2) the syntactic site N_1 is not projected onto the same thematic node. N_1 of (1) is projected onto the same thematic node as N_2 in (3) and N_1 of (2) onto the same thematic node as N_1 of (3). Moreover, how can we calculate the difference between (1) and (2)? Only by the fact that the entity which takes the N_1 place in the common frame is [+human] for (1). By this, we want to mean that a semantic feature ([+ human]) has a syntactic role, because its presence is required for selecting the interpretation where *Pierre* is the "source" and not the "object" of *to steal*. This leads us to distinguish two kinds of semantic features (cf Bès & Lecomte, 1995) in a lexicon. One sort is used in order to select a mapping between a syntactic frame and a thematic structure, the other one is used after a thematic structured has been selected in order to make this thematic structure work and produce a semantic representation as a final result. The first sort features could be called "syntactic", and the second sort "intrinsically semantic". We finally assume that the french verb *voler* has (at least) three different syntactic frames:

(4) N_0 V N_1[+hum]

(5) N_0 V N_1

(6) N_0 V N_1 à N_2

In order to build an interpretation of these verbal constructions, we have to map them onto a common thematic structure, which contains nodes and relations between them. Let us call X, Y, Z the three nodes in the thematic structure. We can describe the semantic of *voler* by saying that it contains a *transfer*-relation between two states: in the initial state, the object Z belongs to Y, in the final state, it belongs to X, and there is also a relation @, referred as *intentional control* (Descles, 1990), from X to Y. In the three frames, N_0 is projected onto X, in (4), N_1 is projected onto Z, in (5) onto Y, in (6), N_1 is projected onto Y and N_2 onto Z. We consider an *agent* as the first argument of the relation @, an *object* as its second argument.
This provides us with the background of our study. It is then necessary to show how we can effectively calculate semantic interpretations on this ground, that means, how we can describe:

- the correct selection of a syntactic frame,
- the production of a semantic formula on the basis of a pairing between the syntactic frame and a thematic structure.

Moreover, we have to take ambiguity cases into account (like the case where (1) must in fact be interpreted in a way similar to (2)), and also metaphoric and metonymic interpretations, phenomena which are usually dealt with by means of a coercion mechanism (Pustejovsky, 1991). In what follows, we shall see that different interpretations come from different *proofs*, and that productions of metaphoric and metonymic interpretations come from a backchaining mechanism that re-instatiates some variables when the deduction process comes to a failure.

3 Producing Meaning as Resource Consumption

We shall assume every verbal entry unfolded into primitive schemes involving typed variables X, Y, Z, ... for marking thematic roles. Schemes are formulae built from atomic ones consisting in an assignment of type to a term. Terms are built from n-ary typed functors, like: movable (1-ary), loc, cause, possess, ... (binary), @, transfer ... (ternary) and so on. For instance, the functor @ takes as arguments two individuals (type **t**) and a state (type **s**) and gives an *event*, that means an entity of type **e**. The functor loc takes two arguments of type **t** and gives a result of type **s**. A unary functor like movable takes an argument of type **t** and gives back a result of the same type. The primitive sign for type assignment is the ":".

By N $\leadsto_\tau$ X we mean that a syntactic site N (in other approaches, it could be a *f-structure* associated with a node in a syntactic tree) *projects onto* the variable X of type τ. In what follows, and for sake of simplicity we limit ourselves to the selection of a head in any syntactic site, but in more elaborate versions, we shall be able to select the head, the specifier or any representation of the complete constituent.

We assume a linear logic formula associated with each verbal entry. This formula contains two parts: one part concerns the necessary conditions for matching a syntactic frame and a thematic structure, the other one concerns a certain amount of semantical informations which will have to be consumed in order to produce an atomic ground semantical formula as a final result. We shall distinguish the two parts by putting the first one in bold characters. Two kinds of variables are used: a first kind, in uppercase letters (X, Y, Z, ...), consists in variables associated with thematic nodes in the structure and must be instantiated by the content of the syntactic sites. A second kind, in lowercase letters (x, y, u, v, ...) will serve for *internal* purposes: they are instantiated by terms resulting from the application of functors during the deduction process.

Let us take as an example the following constructions, using the french verb *rassembler* (Fradin, 1990).

(7) *Pierre rassemble les habits* (Pierre gathers the clothes together)

(8) *l'armoire rassemble les habits* (the cupboard keeps all the clothes together)

(9) *Pierre rassemble les habits dans l'armoire* (Pierre gathers the clothes together in the cupboard)

The verb *rassembler* has a thematic structure which contains three variables: X and Y of type **t**, Z of type **s**. X must be interpreted as an agent, Y as an object (or a theme) and Z as a location. But there are at least three syntactic frames corresponding to respectively (7), (8) and (9):

(10) N_0[+hum] V N_1

(11) N_0 V N_1

(12) N_0 V N_1 prep-loc N_2.

The thematic structure must contain the facts that when Y is instantiated, it must be by a movable entity, when Z is instantiated, it must be by a state where the movable Y is localized, and when X is instantiated, it must be by an agent in the relation @, the other arguments of which being the movable Y and the localisation of Y in Z. What we have to show in every case is that all these conditions are used in some deduction of a formula which will express in addition the type of the result (for instance **e** for (7) and (9) and **s** for (8)).
We use the following formula:

$$
\begin{array}{l}
\forall X, Y, Z \\
(((\mathbf{human}(\mathbf{X}) \otimes (\mathbf{N_0} \rightsquigarrow_{\mathbf{t}} \mathbf{X})) \otimes (\mathbf{N_1} \rightsquigarrow_{\mathbf{t}} \mathbf{Y}) \otimes ((\mathbf{N_2} \rightsquigarrow_{\mathbf{t}} \mathbf{Z}) \oplus \mathbf{1}))) \qquad \oplus \\
((\mathbf{N_0} \rightsquigarrow_{\mathbf{s}} \mathbf{Z}) \otimes (\mathbf{N_1} \rightsquigarrow_{\mathbf{t}} \mathbf{Y})) \multimap \\
\qquad (\forall u, w(\mathsf{loc}(u, w) : s) \multimap (@(X, u, \mathsf{loc}(u, w)) : e))\&\mathbf{1} \\
\qquad \otimes(\mathsf{movable}(Y) \multimap (\mathsf{movable}(Y) : t)) \\
\qquad \otimes(\forall v(\mathsf{movable}(v) : t) \otimes \mathsf{in}(v, Z) \multimap (\mathsf{loc}(\mathsf{movable}(v), Z) : s)) \\
\qquad \&(\forall v(\mathsf{movable}(v) : t) \multimap \exists T \mathsf{in}(v, T) \otimes (\mathsf{loc}(\mathsf{movable}(v), T) : s))
\end{array}
$$

Let us assume the lexicon contains:

$$!\mathrm{human}(Pierre),\ !\mathrm{movable}(habits)$$

A deduction for (7) is:

$$
\begin{array}{lll}
(1) & !human(Pierre) & [lexicon] \\
(2) & N_0 \rightsquigarrow_{\mathbf{t}} Pierre & [sentence] \\
(3) & N_1 \rightsquigarrow_{\mathbf{t}} habits & [sentence] \\
(4) & \mathbf{1} & [i\mathbf{1}] \\
(5) & (N_2 \rightsquigarrow_{\mathbf{t}} Z) \oplus \mathbf{1} & [i\oplus] \\
(6) & human(Pierre) \otimes (N_0 \rightsquigarrow_{\mathbf{t}} Pierre)) \otimes & \\
 & (N_1 \rightsquigarrow_{\mathbf{t}} habits) \otimes ((N_2 \rightsquigarrow_{\mathbf{t}} Z) \oplus \mathbf{1}) & [i\otimes] \\
(7) & (human(Pierre) \otimes (N_0 \rightsquigarrow_{\mathbf{t}} Pierre) \otimes & \\
 & (N_1 \rightsquigarrow_{\mathbf{t}} habits) \otimes ((N_2 \rightsquigarrow_{\mathbf{t}} Z) \oplus \mathbf{1})) & \\
 & \oplus((N_0 \rightsquigarrow_{\mathbf{s}} Z) \otimes (N_1 \rightsquigarrow_{\mathbf{t}} habits)) & [i\oplus] \\
(8) & (\forall u, w(\mathsf{loc}(u, w) : s) \multimap & \\
 & (@(Pierre, u, \mathsf{loc}(u, w)) : e))\&\mathbf{1} & \\
 & \otimes(\mathsf{movable}(habits) \multimap (\mathsf{movable}(habits) : t)) & \\
 & \otimes(\forall v(\mathsf{movable}(v) : t) \otimes \mathsf{in}(v, Z) \multimap & \\
 & (\mathsf{loc}(\mathsf{movable}(v), Z) : s)) & \\
 & \&(\forall v(\mathsf{movable}(v) : t) \multimap \exists T \mathsf{in}(v, T) & \\
 & \otimes(\mathsf{loc}(\mathsf{movable}(v), T) : s)) & [e\forall][e \multimap] \\
(9) & \mathsf{movable}(habits) \multimap (\mathsf{movable}(habits) : t) & [e\otimes]
\end{array}
$$

(10)	$!\mathsf{movable}(habits)$	$[lexicon]$
(11)	$(\mathsf{movable}(habits) : t)$	$[e \multimap]$
(12)	$(\forall v(\mathsf{movable}(v) : t) \otimes \mathsf{in}(v, Z) \multimap$ $(\mathsf{loc}(\mathsf{movable}(v), Z) : s))$ $\&(\forall v(\mathsf{movable}(v) : t) \multimap \exists T \mathsf{in}(v, T)$ $\otimes(\mathsf{loc}(\mathsf{movable}(v), T) : s))$	$[e\otimes]$
(13)	$(\forall v(\mathsf{movable}(v) : t) \multimap \exists T \mathsf{in}(v, T)$ $\otimes(\mathsf{loc}(\mathsf{movable}(v), T) : s))$	$[e\&]$
(14)	$(\mathsf{movable}(habits) : t) \multimap \exists T \mathsf{in}(habits, T)$ $\otimes(\mathsf{loc}(\mathsf{movable}(habits), T) : s)$	$[e\forall]$
(15)	$\exists T \mathsf{in}(habits, T) \otimes (\mathsf{loc}(\mathsf{movable}(habits), T) : s)$	$[e \multimap]$
(16)	$(\forall u, w(\mathsf{loc}(u, w) : s) \multimap (@(Pierre, u, \mathsf{loc}(u, w)) : e))\&\mathbf{1}$	$[e\otimes]$
(17)	$((\mathsf{loc}(\mathsf{movable}(habits), T) : s) \multimap$ $(@(Pierre, \mathsf{movable}(habits), \mathsf{loc}(\mathsf{movable}(habits), T)) : e))\&\mathbf{1}$	$[e\forall]$
(18)	$((\mathsf{loc}(\mathsf{movable}(habits), T) : s) \multimap$ $(@(Pierre, \mathsf{movable}(habits), \mathsf{loc}(\mathsf{movable}(habits), T)) : e))$	$[e\&]$
(19)	$\exists T \mathsf{in}(habits, T)\otimes$ $(@(Pierre, \mathsf{movable}(habits), \mathsf{loc}(\mathsf{movable}(habits), T)) : e))$	$[e \multimap]$

Comments:
This deduction begins with premises which are provided by the sentence: (2) and (3). These are considered as resources which have to be consumed once. The lexicon provides two resources: (1) and (10) which can be consumed any number of times. These resources are necessary in order the deduction proceeds. The resource (1) allows the first alternative be chosen in (7). (7) is right because it needs only the first disjunct in order to be true. In (4), we introduce **1** according to the allowance of introducing this neutral element anywhere. It makes the additive disjunction in (5) true, and therefore (6), and then (7). By modus ponens, we get (8), which is a multiplicative conjunction of several resources (in fact semantic rules). Each conjunct may be separately used, by [e $\otimes$] as it can be seen in (9), (12) and (16). For sake of brievety, two steps are made in (8): one is elimination of $\forall$ and the other is modus ponens. The resource (9) can be used because of the lexical fact (10). That gives (11). But the resource in (11) is used by means of the rule stated in (12). This rule gives us two choices: one choice is relative to the case where Z is instantiated, the other to the case where it is not. Actually, it is not. We thus choose the alternative stated in (13). By modus ponens, it gives (15). A non mentionned step is elimination of $\exists$, (and after, its re-introduction). It would be done by taking an arbitrary τ and then saying: (*)in(habits, τ) $\otimes$ (loc(movable(habits), τ): s). By elimination of $\otimes$ in this formula, we get the the resource which has to be consumed in order to obtain (@(Pierre, movable(habits), loc(movable(habits), τ)):e)), by the choice of the first alternative in (17) (step 18). But the first conjunct of (*) is still there, and by introduction of $\exists$ and introduction of $\otimes$, we get (19).
A deduction will be said *fair* if and only if:

- all syntactic resources are used (resources of the sort: $N_0 \leadsto_{\mathbf{t}}$ *Pierre*),
- all selected rules (= linear implications) are consumed ("selected" rules means: rules that are chosen according to certain choices stated in the formula assiocociated with the verb, in order to consume the syntactic resources),
- the only lexical facts that can be used in a deduction are ground facts: there is no lexical fact involving variables (this, for instance, constrains us choose the second alternative in (12), because we shall never meet a fact in(v, Z) if Z is not instantiated)

A sentence will be said *semantically correct* if and only if it gives raise to (at least) one fair deduction.
We now can show, as a second example, that the sentence *l'armoire rassemble les habits* is also semantically correct. Let us notice before beginning that the solution will be similar to the solution we could have taken if we had not used the resource human(Pierre) in the previous example. The meaning would have then been the meaning of: *Pierre keeps all the clothes together*(on himself).

(1)	$N_0 \leadsto_{\mathbf{t}}$ *armoire*	$[sentence]$
(2)	$N_1 \leadsto_{\mathbf{t}}$ *habits*	$[sentence]$
(3)	$(human(X) \otimes (N_0 \leadsto_{\mathbf{t}} armoire) \otimes$ $(N_1 \leadsto_{\mathbf{t}} habits) \otimes ((N_2 \leadsto_{\mathbf{t}} armoire) \oplus \mathbf{1}))$ $\oplus((N_0 \leadsto_{\mathbf{s}} armoire) \otimes (N_1 \leadsto_{\mathbf{t}} habits))$	$[i\oplus]$
(4)	$(\forall u, w(\mathsf{loc}(u,w) : s) \multimap$ $(@(Pierre, u, \mathsf{loc}(u,w)) : e))\&\mathbf{1}$ $\otimes(\mathsf{movable}(habits) \multimap (\mathsf{movable}(habits) : t))$ $\otimes(\forall v(\mathsf{movable}(v) : t) \otimes \mathsf{in}(v, Z) \multimap$ $(\mathsf{loc}(\mathsf{movable}(v), Z) : s))$ $\&(\forall v(\mathsf{movable}(v) : t) \multimap \exists T \mathsf{in}(v, T)$ $\otimes(\mathsf{loc}(\mathsf{movable}(v), T) : s))$	$[e\forall][e \multimap]$
(5)	$\mathsf{movable}(habits) \multimap (\mathsf{movable}(habits) : t)$	$[e\otimes]$
(6)	$!\mathsf{movable}(habits)$	$[lexicon]$
(7)	$(\mathsf{movable}(habits) : t)$	$[e \multimap]$
(8)	$(\forall v(\mathsf{movable}(v) : t) \otimes \mathsf{in}(v, Z) \multimap$ $(\mathsf{loc}(\mathsf{movable}(v), Z) : s))$ $\&(\forall v(\mathsf{movable}(v) : t) \multimap \exists T \mathsf{in}(v, T)$ $\otimes(\mathsf{loc}(\mathsf{movable}(v), T) : s))$	$[e\otimes]$
(9)	$(\forall v(\mathsf{movable}(v) : t) \otimes \mathsf{in}(v, Z) \multimap$ $(\mathsf{loc}(\mathsf{movable}(v), Z) : s))$	$[e\&]$
(10)	$(\mathsf{movable}(habits) : t) \otimes \mathsf{in}(habits, armoire) \multimap$ $(\mathsf{loc}(\mathsf{movable}(habits), armoire) : s)$	$[e\forall]$
(11)	$\mathsf{in}(habits, armoire)$	$[lexicon]$
(12)	$(\mathsf{movable}(habits) : t) \otimes \mathsf{in}(habits, armoire)$	$[i\otimes]$
(13)	$(\mathsf{loc}(\mathsf{movable}(habits), armoire) : s)$	$[e \multimap]$

As we can see, while the former deduction gave us a result of type e (*event*), the latter gives us a result of type s (*state*). We let to the reader the task to derive a meaning in a fair deduction for the sentence *Pierre rassemble les habits dans l'armoire*.

4 Conclusion

The aim of this paper was to show how we can deal with semantic resources in a framework like linear logic. We have used in fact the Intuitionnistic Linear Logic: it is the reason why it was easy to represent the deductions in a Natural Deduction Form. (Let us recall that Intuitionnistic Logic differs from Classical by the fact that only one conclusion is searched for a given proof, proofs are never multi-conclusions in Intuitionnistic Logic). Advantages of Linear Logic are obvious: if compared with classical logic, the approach can be based on a conception of resource-consumption (which is not possible in classical logic), if compared with λ-calculus, no ordering is imposed on resources. Additive connectives may be used, thus providing a convenient way to express choices of two kinds: external choices ($\oplus$) express the various combinations of syntactic datas (premises), and internal choices ($\&$) express what we can choose when we are guided by a conclusion. The neutral element **1** when combined by & to some formula F in an available resource allows to discard F. We do not extend here the analysis of coercion: that will be done in another paper, but it is worth to notice that the lexical facts can also be stated as choices. Each conjunct (of &) can be used alternatively in order to achieve a fair deduction. For example, in a sentence like *Pierre a rassemblé sur une page tous les présidents français* (Pierre gathered all the French presidents on one page), no predicate in(président, page) is available! But fortunately, a predicate in(word, page) is available, and we can assume the entry for *president* contains something like:

$$\textit{president} : \text{human}(\textit{president}) \ \& \ \text{word}(\text{name\_of } \textit{president})$$

Then, when meeting a failure with the first alternative (human(*president*)), the second is used, and leads to an instanciation of Y not by *president* but by name_of *president*, thus realizing the coercion mechanism.